Math 3A
Winter 2016
Midterm 1
1/29/2016
Time Limit: 50 Minutes

Name (Print): ____________________

Student ID ____________________

This exam contains 7 pages (including this cover page) and 3 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a "theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	50	
2	50	
3	50	
Total:	150	

1. Let

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \text{ and } \mathbf{b} = \begin{pmatrix} e \\ f \end{pmatrix}. \tag{1}$$

(a) (40 points) Suppose that $a \neq 0$, compute the solution of $\mathbf{Ax} = \mathbf{b}$ using row reduction and provide the conditions on a, b, c, d such that your computations are valid. (**Hint:** remember that you can not divide by zero.)

(b) (5 points) If $a = 0$, and $c \neq 0$, is your above computation still valid? How would you modify it? (explain briefly) (**Hint:** remember that you can swap the equations and the result is the same.)

(c) (5 points) If $a = 0$, $c = 0$, but $b \neq 0$, $d \neq 0$, what are the conditions on e and f such that the system $\mathbf{Ax} = \mathbf{b}$ has a solution? Is the solution unique? (**Hint:** remember that $\mathbf{Ax} = \mathbf{b}$ has a solution if and only if $\mathbf{b}$ can be written as a linear combination of the columns of $\mathbf{A}$.)

2. Consider the transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by

$$T(\mathbf{x}) = \begin{pmatrix} x_2 - x_1 \\ x_1 + x_2 \\ hx_3 + q \end{pmatrix}, \text{ where } \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}; \tag{2}$$

in which, h and q are real numbers.

(a) (5 points) What is the condition on q such that the transformation is linear? (**Hint:** remember that if T is linear then $T(c\mathbf{x}) = cT(\mathbf{x})$, which is valid for $c = 0$.)

(b) (10 points) **From now we suppose that** $q = 0$. Write the associated matrix $\mathbf{A}$ of the transformation T. (**Hint:** remember that $\mathbf{A}(:, i) = T(\mathbf{e}_i)$.)

(c) (10 points) What is the condition on h such that the transformation T is **NOT** one-to-one? Explain briefly.

(d) (5 points) What is the condition on h such that the transformation T is **NOT** onto $\mathbb{R}^3$? Explain briefly.

(e) (5 points) **Suppose that** $h = 0$, then

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \text{ and let } \mathbf{b} = \begin{pmatrix} 0 \\ 2 \\ r \end{pmatrix}. \tag{3}$$

What is the condition on r such that the system $\mathbf{Ax} = \mathbf{b}$ has a solution? Is the solution unique? (**Hint:** remember to check that the system is consistent.)

(f) (15 points) **Suppose that** $h = 0$ **and** $r = 0$. Solve $\mathbf{Ax} = \mathbf{b}$ and give the answer in parametric form.

3. (50 points) Write a small Matlab code that performs the following operations.

   (a) (22 points) Write using the C convention a matrix-vector multiplication, $\mathbf{y} = \mathbf{A}\mathbf{x}$ for a matrix $\mathbf{A} \in \mathbb{R}^{m \times n}$ and a vector $\mathbf{x} \in \mathbb{R}^n$. The inputs are $\mathbf{A}$ and $\mathbf{x}$ and the output is $\mathbf{y}$

   (b) (22 points) Write using the Fortran convention a matrix-vector multiplication, $\mathbf{y} = \mathbf{A}\mathbf{x}$ for a matrix $\mathbf{A} \in \mathbb{R}^{m \times n}$ and a vector $\mathbf{x} \in \mathbb{R}^n$. The inputs are $\mathbf{A}$ and $\mathbf{x}$ and the output is $\mathbf{y}$

   (c) (6 points) Knowing the MATLAB uses the Fortran convention to store matrices, which algorithm will run faster if implemented in MATLAB? and why?