

Math 2D
Winter 2017
Midterm 1
02/06/2017

Name (Print): _____

Time Limit: 50 Minutes

Student ID _____

This exam contains 10 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a “theorem” you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	10	
2	10	
3	20	
4	15	
5	45	
Total:	100	

Do not write in the table to the right.

1. Find the distance between the spheres

(a) (5 points)

$$x^2 + y^2 + z^2 = 4,$$

and

$$x^2 + y^2 + z^2 + 2x + 4y + 6z - 86 = 0.$$

(b) (5 points)

$$x^2 + y^2 + z^2 = 9,$$

and

$$x^2 + y^2 + z^2 - 10x - 8y - 6z - 50 = 0.$$

2. Find the distance from the origin to the lines given by

(a) (5 points)

$$x = 2t + 1; y = t - 1; z = t - 5; \quad t \in \mathbb{R}.$$

(b) (5 points)

$$x = 4t + 2; y = -2t - 1; z = 6t + 3; \quad t \in \mathbb{R}.$$

3. Find dy/dx and d^2y/dx^2 of the following curves

(a) (10 points)

$$x = t + \sin t; y = t - \cos t.$$

(b) (10 points)

$$x = 1 + t^3, y = t - \log_{10} t.$$

4. (15 points) Use the properties of the dot and cross product to show that if $\mathbf{a}, \mathbf{b}, \mathbf{c} \in \mathbb{R}^3$ then

$$(\mathbf{a} \times \mathbf{b}) \cdot [(\mathbf{b} \times \mathbf{c}) \times (\mathbf{c} \times \mathbf{a})] = [\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})]^2.$$

5. A curve called the folium of Descartes is defined by the following parametric equations

$$x = \frac{3t}{1+t^3}, \quad y = \frac{3t^2}{1+t^3}.$$

- (a) (10 points) Show that if a point (a, b) lies on the curve, then so does (b, a) . What is the symmetry of the curve?

- (b) (5 points) In which points does the curve intersects the line $x = y$.

(c) (10 points) Find the points on the curve in which the tangent lines are horizontal or vertical.

(d) (5 points) Sketch the curve (you can use that $y = -x - 1$ is a slant asymptote)

(e) (5 points) Show that the points on the curve satisfy

$$x^3 + y^3 = 3xy$$

(f) (10 points) Show that the polar equation of the curve is given by

$$r = \frac{3 \sec \theta \tan \theta}{1 + \tan^3 \theta}.$$