Math 2B
Spring 2017
Midterm 1
04/26/2017
Time Limit: 50 Minutes

Name (Print): ______________________________

Student ID ______________________________

This exam contains 10 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a "theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	15	
2	12	
3	20	
4	18	
5	14	
Total:	79	

1. Determine if the following statements are true or false. For each case explain your answers.

   (a) (3 points) The function defined by

   $$f(x) = \begin{cases} \frac{\sin(x)}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

   is integrable in the interval $[-1, 1]$

   (b) (3 points)

   $$\int_{-1}^{1} \left[x^5 + \cos(x)\sin(x)\right] dx = 0$$

   (c) (3 points)

   $$\int_{0}^{\pi/4} (x+1)^2 \cos^2(x) dx = 0$$

(d) (3 points) If a continuous function f satisfies

$$\int_0^1 f(x)dx = 0$$

then $f(x) = 0$ for $x \in [0,1]$.

(e) (3 points)

$$\lim_{x\to 0} \frac{\int_0^x \sin(t)dt}{x^2} = \frac{1}{2}$$

2. (a) (6 points) Use three rectangles and the midpoint rule to approximate the area under the graph $f(x) = \frac{1}{3}x + 1$ and above the x-axis from $x = 1$ to $x = 7$.

(b) (6 points) Find an expression for the same area as a limit of a Riemann sum.

3. Evaluate the following expressions.

   (a) (4 points) $\int \cos^2(x)\,dx$

   Hint: use the half-angle formula $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$.

   (b) (4 points) $\int_{\pi/4}^{\pi/2} (\cos^2(y) - \sin^2(y))\,dy$

   Hint: You may want to use the identity $\sin^2(x) = 1 - \cos^2(x)$, and the result of the question a).

(c) (7 points) $\displaystyle\int_0^1 \frac{x^3}{(1+x^4)^5}\,dx$

(d) (5 points) $\displaystyle\frac{d}{dx}\left(\int_{1/x}^{\sin^2(3x)} \frac{1}{\sqrt{t^2+1}}\,dt\right)$

4. Let $\mathcal{R}$ be the region bounded by $y = \sqrt{x}$ and $y = x/2$.

   (a) (9 points) Compute the area of $\mathcal{R}$.

   (b) (9 points) Setup the integral to compute the volume obtained by revolving the region $\mathcal{R}$ about the line $y = -2$. (You do not need to compute the integral)

5. The acceleration function (in meters per second squared) for a particle moving along a line is given by $a(t) = 3t^2 - 6t + 3$ for $t \in [0, 2]$.

   (a) (5 points) Verify that the velocity function, when $v(0) = -1$, is given by $v(t) = t^3 - 3t^2 + 3t - 1$.

(b) (4 points) Find the the displacement between time $t = 0$ and $t = 2$.

Hint 1: the displacement is given by $\int_a^b v(t)\,dt$.

Hint 2: to compute the integral you may find that factorizing the velocity using that $(x-1)^3 = x^3 - 3x^2 + 3x - 1$, and a well chosen substitution may make your computations easier.

(c) (5 points) Find the distance traveled by the particle during the given time interval.

Hint 1: the total distance traveled is given by $\int_a^b |v(t)|\, dt$.

Hint 2: You will need to plot $v(t)$, and find the different intervals in which the function $v(t)$ has constant sign.