Practice Final 2

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1. Compute the fourth Riemann Sum for $f(x) = e^x + \arctan(x) + 2$ from $x = -1$ to $x = 1$. Use right endpoints as your sample points.

2. Let $f(x)$ be continuous and $\int_0^{16} f(x)\mathrm{d}x = 6$. Find $\int_0^4 xf(x^2)\mathrm{d}x$.

3. Compute $\int_{-2}^{3} \left(x^2 - 4x + 3\right) \mathrm{d}x$ by evaluating the limit of its Riemann sums.

4. Suppose a particle moves back and forth along a straight line with velocity $v(t)$, measured in feet per second, and and acceleration $a(t)$. What are the meanings of $\int_0^{60} v(t)\mathrm{d}t$, $\int_0^{60} |v(t)|\mathrm{d}t$ and $\int_0^{60} a(t)\mathrm{d}t$?

5. (a) Simplify $\frac{d}{dx}\left[\int_{x^2}^{10} (\arctan(t) - 4)\,\mathrm{d}t\right]$

(b) Given $h(2) = 1$, $h(5) = 8$, $h'(2) = 2$, $h'(5) = 7$. Evaluate $\int_2^5 xh''(x)\mathrm{d}x$.

6. (a) Find the most general antiderivative of $x^2 - 4x + 3$.

(b) Evaluate $\int_{-1}^{3} \left(x^2 - 4x + 3\right) \mathrm{d}x$

7. Suppose $g(-2) = 3$, $g(2) = 9$ and $\int_{36}^{108} f(z)\mathrm{d}z = 15$. Evaluate $\int_{-2}^{2} 4f\left(12g(x)\right) g'(x)\mathrm{d}x$.

8. Evaluate the integral $\int_0^1 \frac{1}{(5x+2)^{30}}\mathrm{d}x$. Show all work.

9. Evaluate the integral $\int \tan(\frac{x}{2})\mathrm{d}x$. Show all work.

10. Evaluate the integral $\int_{e}^{e^2} \ln(x)\mathrm{d}x$. Show all work.

11. Evaluate the integral $\int \arctan(3x)\,\mathrm{d}x$. Show all work.

12. Evaluate the integral $\int_{-1}^{3} \frac{x^3}{\sqrt{36-x^2}}\mathrm{d}x$. Show all work.

13. Evaluate the integral $\int \sqrt{4x^2+25}\mathrm{d}x$. Show all work.

14. Evaluate the integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan^2(x)\mathrm{d}x$. Show all work.

15. Evaluate the integral $\int \sin^2(x)\cos^4(x)\mathrm{d}x$. Show all work.

16. Evaluate the integral $\int_0^1 \frac{4x}{x^3+x^2+x+1}\mathrm{d}x$. Show all work.

17. Evaluate the integral $\int \frac{6x^2+8x+3}{x^3+x}\mathrm{d}x$. Show all work.

18.Determine whether the improper integral $\int_0^\infty \cos(x)\mathrm{d}x$ is convergent or divergent. Evaluate the integral if convergent, or explain why it diverges. Show all work.

19. Determine whether the improper integral $\int_{-2}^{0} \frac{1}{x^2+5x+6} \mathrm{d}x$ is convergent or divergent. Evaluate the integral if convergent, or explain why it diverges. Show all work.

20. A particle moves along a line with velocity function $v(t) = t^3 - 7t^2 + 10t$, where v is measured in meters per second. Find (a) the displacement and (b) the distance traveled by the particle during the time interval $[0, 3]$.

21-22. Let R be the region bounded by the graphs of $y = 4 - x^2$ and $y = 0$.
(a) Sketch the region R and find its area.

(b) Set up an integral to compute the volume of the solid whose cross-sections perpendicular to x-axis are equilateral triangles. Do not evaluate the integral!

23. Find the exact length of the curve $y = 2\ln\left(\sin\frac{1}{2}x\right)$ for $\frac{\pi}{3} \le x \le \pi$.

24. Find the average of the function $f(x) = x^{-2}e^{\frac{1}{x}}$ over the interval $[1, 3]$. Show all work.

25. Using integration find the area of the triangle with vertices $A = (-2, 7)$, $B = (13, 4)$, $C = (4, -5)$ and sides AB : $x + 5y = 33$, $BC : x - y = 9$, $CA : 2x + y = 3$.

26. For the sequence $a_n = \frac{-n^3}{n^2+1}$ determine if the sequence is (a) monotone, (b) bounded, and (c) what conclusion can you make based on (a) and (b)?

27. Use the Squeeze Theorem to show that the sequence $b_n = \frac{(-1)^{n+1}}{n^2}$ converges.

28. Determine the general term formula for the sequence $\left\{2, -3, \frac{9}{2}, -\frac{27}{4}, \frac{81}{8} \ldots\right\}$. Use the formula to find the 50^{th} term.

29-31. For each of the following sequences $\{b_n\}_{n=1}^{\infty}$, compute the $\lim_{n\to\infty} b_n$. If a limit doesn't exist, expain why not. Show all work.

(a) $b_n = e^{-n} \cdot n$

(b) $b_n = \cos\left(\frac{\pi n}{4}\right)$

(c) $b_n = \frac{1}{n}$

32. Find the sum $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$.

33. Use the Divergence Test to determine that the series $\sum_{n=1}^{\infty} (-1)^n \cos(\pi n)$ diverges. Show all work.

34. Use the Alternating Series Test to determine whether the series $\sum_{n=1}^{\infty} \frac{n\cos(\pi n)}{n^5+1}$ is convergent or divergent. Show all work.

35. Use the Direct or Limit Comparison Test to determine whether the series $\sum_{n=1}^{\infty} \frac{n^2-1}{n^3-n-2}$ is convergent or divergent. Show all work.

36. Use the Ratio or Root Test to determine whether the following series is convergent or divergent. Show all work.

(a) $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$

(b) $\sum_{n=1}^{\infty} \frac{2^n(-1)^n}{n^n}$

37. Use any Convergence Test to determine whether the following series is convergent or divergent. Show all work.

(a) $\sum_{n=3}^{\infty} \frac{1}{n \ln n}$

(b) $\sum_{n=2}^{\infty} \frac{\ln n}{n^3-5}$

(c) $\sum_{n=1}^{\infty} \frac{n!}{2^n}$

38. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{\sqrt[3]{n}}$.

39. Find the first three nonzero terms of the Taylor series expansion of $f(x) = \sin^2(x)$ about $x = \pi$.

40. Find the Maclaurin series for $f(x) = x\cos(x)$.