Practice Final 1

by Nikki Fider and Anton Butenko

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1. Estimate the area under $f(x) = \sin(x)$ on the interval $[0, \pi]$ by computing the Riemann sum using three subintervals and left endpoints.

2. Evaluate the integral $\int_{-2\pi}^{2\pi} \sqrt{4\pi^2 - x^2} \sin(x) \, \mathrm{d}x$. Show all work.

3. Write the integral $\int_5^{10} (6x + \cos(x) - 1) \, \mathrm{d}x$ as a limit of Riemann sums using right endpoints..

4. A function for the basal metabolism rate, in kcal/h, of a young man is $R(t)$, where t is the time in hours measured from $5:00$ AM. What does the integral $\int_0^{24} R(t)\mathrm{d}t$ represent? What are the units?

5. (a) Given $\int_1^{-3} h(x)\mathrm{d}x = 2$, $\int_{-3}^{1} 3f(t)\mathrm{d}t = 6$. Evaluate $\int_{-3}^{1} \left(4f(z) - \frac{1}{2}h(z)\right) \mathrm{d}z$.

(b) Let $h(x) = \int_{e^x+x}^{x \ln x} t\mathrm{d}t$. Find $h'(x)$.

6. Evaluate $\int_0^1 \sqrt{v}\left(v^3+2\right)^2 \mathrm{d}v$

7. Suppose $\int_1^{e^2} f(z)\mathrm{d}z = 10$. Evaluate $\int_0^1 e^{2x} f\left(e^{2x}\right) \mathrm{d}x$.

8. Evaluate the integral $\int_1^2 \frac{x^{10}}{1+x^{22}} \mathrm{d}x$. Show all work.

9. Evaluate the integral $\int \frac{\ln\left(e^{x \ln x}\right)}{x^2} \mathrm{d}x$. Show all work.

10. Evaluate the integral $\int_0^{\frac{\pi}{2}} 3x^2 \cos(x)\mathrm{d}x$. Show all work.

11. Evaluate the integral $\int e^{2x} \sin(\pi x)\mathrm{d}x$. Show all work.

12. Evaluate the integral $\int_0^1 \frac{1}{(x^2+1)^2}\mathrm{d}x$. Show all work.

13. Evaluate the integral $\int \frac{x^2}{\sqrt{1-9x^2}}\mathrm{d}x$. Show all work.

14. Evaluate the integral $\int_0^{\frac{\pi}{2}} \sin^3(x)\cos^3(x)\mathrm{d}x$. Show all work.

15. Evaluate the integral $\int \sec^{20}(x)\tan^5(x)\mathrm{d}x$. Show all work.

16. Evaluate the integral $\int_0^1 \frac{2}{2x^2+3x+1} \mathrm{d}x$. Show all work.

17. Evaluate the integral $\int \frac{x^5+x-1}{x^3+1} \mathrm{d}x$. Show all work.

18. Determine whether the improper integral $\int_0^\infty re^{-3r}dr$ is convergent or divergent. Evaluate the integral if convergent, or explain why it diverges. Show all work.

19. Determine whether the improper integral $\int_0^{\frac{\pi}{2}} \sec^2 \theta d\theta$ is convergent or divergent. Evaluate the integral if convergent, or explain why it diverges. Show all work.

20. A particle moves along a line with velocity function $v(t) = \cos t$, where v is measured in feet per hour. Find (a) the displacement and (b) the distance traveled by the particle during the time interval $[0, \frac{2\pi}{3}]$.

21-22. Let R be the region bounded by the graphs of $x = 2y^2$ and $x = 4 + y^2$.
(a) Sketch the region R and find its area.

(b) Set up an integral to compute the volume of the solid generated by revolving the region R (from part (a)) about the y-axis. Do not evaluate the integral!

(c) Set up an integral to compute the volume of the solid generated by revolving the region R (from part (a)) about the line $x = -1$. Do not evaluate the integral!

23. Find the exact length of the curve $y = \frac{1}{4}x^2 - \ln\sqrt{x}$ for $1 \leq x \leq 2$.

24. Find the average of the function $f(x) = \frac{x^4+x^2+\ln(e^x)}{x^2}$ over the interval $[2,3]$. Show all work.

25. Using integration find the area of the triangle with vertices $A = (-2,-4)$, $B = (1,5)$, $C = (10,-1)$ and sides $AB : 3x - y = -2$, $BC : 2x + 3y = 17$, $CA : x - 4y = 14$.

26. For the sequence $b_n = n^{-1}\sin\left(\frac{\pi}{2n}\right)$ determine if the sequence is (a) monotone, (b) bounded, and (c) what conclusion can you make based on (a) and (b)?

27. Use the Squeeze Theorem to show that the sequence $c_n = \frac{4+\sin(n)}{3n+1}$ converges.

28. Determine the general term formula for the sequence $\left\{\frac{1}{10}, \frac{1}{15}, \frac{1}{20}, \frac{1}{25}, \frac{1}{30} \ldots\right\}$. Use the formula to find the 100^{th} term.

For each of the following sequences $\{a_n\}_{n=1}^{\infty}$, compute the $\lim_{n\to\infty} a_n$. If a limit doesn't exist, expain why not. Show all work.

29. $a_n = (-2)^n$

30. $a_n = \arctan\left(\frac{n^5+4}{1-n^3}\right)$

31. $a_n = \sin(2\pi n)$

32. Find the sum $\sum_{n=2}^{\infty} \frac{3^n+5^n}{7^{n+1}}$

33. Use the Divergence Test to determine that the series $\sum_{n=1}^{\infty} \arctan\left(\frac{1-n^2}{n}\right)$ is divergent. Show all work.

34. Use the Alternating Series Test to determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{4n}$ is convergent or divergent. Show all work.

35. Use the Direct or Limit Comparison Test to determine whether the series $\sum_{n=1}^{\infty} \frac{5^n}{n3^n}$ is convergent or divergent. Show all work.

36. Use the Ratio or Root Test to determine whether the following series is convergent or divergent. Show all work.

(a) $\sum_{n=1}^{\infty} \frac{(-5)^{2n}}{(n+1)!}$

(b) $\sum_{n=1}^{\infty} \left(\frac{3n}{2n+1}\right)^{5n}$

37. Use any Convergence Test to determine whether the following series is convergent or divergent. Show all work.

(a) $\sum_{n=3}^{\infty} \frac{(\ln n)^{-3}}{n}$

(b) $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$

(c) $\sum_{n=1}^{\infty} \frac{e^n}{n^2+\ln n}$

38. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{3^n (x+4)^n}{\sqrt{n}}$.

39. Find the first three nonzero terms of the Taylor series expansion of $f(x) = \ln(x)$ about $x = e$.

40. Find the Maclaurin series for $f(x) = e^x + e^{2x}$.