1. Compute the derivatives of the following functions and specify the domain of each of them.

   (a)
$$f(x) = \sqrt{x + \sqrt{x + \sqrt{x + \sqrt{x}}}}$$

   (b)
$$f(x) = x^2 \ln\left(\frac{\sin(x)}{x}\right)$$

   (c)
$$f(x) = (x^2+2)^2(x^4+4)^4$$

   (d)
$$f(x) = x^{\sin x}$$

   (e)
$$f(x) = (\sin x)^{\ln x}$$

   (f)
$$f(x) = \frac{\ln x}{1 + \ln x}$$

   (g)
$$f(x) = (\ln x)^{\cos x}$$

   (h)
$$f(x) = (\tan x)^{1/x}$$

   (i)
$$f(x) = \sin(\cos(\tan^{-1} x)))$$

   (j)
$$f(x) = \sin(\ln(x))$$

   (k)
$$f(x) = \sin^{-1}(\sqrt{\sin x})$$

   (l)
$$f(x) = \sqrt{\frac{1+x}{1-x}}$$

   (m)
$$f(x) = \tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$$

   (n)
$$f(x) = \ln\left(\frac{1 + x\sqrt{2} + x^2}{1 - x\sqrt{2} + x^2}\right)$$

2. Find a formula for $f^{(n)}(x)$ if $f(x) = \ln(x-1)$.

3. Find a formula for $f^{(n)}(x)$ if $f(x) = xe^x$. (**Hint:** Show that f satisfies $f' = e^x + f$ and find a recursion formula for $f^{(n)}(x)$)

4. Let g be a twice differentiable function, such that $g'(x) \neq 0$, and $\kappa \in \mathbb{R}$. Then define

$$f(x) = \cos(\kappa g(x))$$

   (a) Show that f satisfies

$$f'' - f'\frac{g''}{g'} + (\kappa g')^2 f = 0$$

   (b) Using the different equation, compute $f^{(n)}(0)$ for $g(x) = x$.

5. Find $f^{(n)}(x)$ of

   (a)

$$f(x) = a\cos(ax)$$

   (b)

$$f(x) = a^x$$

   (c)

$$f(x) = \sin(x)$$

   (d)

$$f(x) = \frac{1-x}{1+x}$$

   (e)

$$f(x) = \sin^2(x)$$

   (f)

$$f(x) = x^{n-1}\ln(x)$$

6. Show that the functions $f(x) = x^2 - \cos(x)$ and $g(x) = 2x^2 - x\sin(x) - \cos^2(x)$ only have two zeros. (**Hint**: Use the Intermediate value Theorem, with carefully chosen points to show that the zeros exist, and use a growth argument to show that they are unique)

7. Find the tangent line of the following curves at the specified points

   (a) The curve given by $e^{2\sin^{-1}(yx)} = \ln(1 + x^2 + y^2)$, at the point in which the curve intersects $y = 0$ such that $x > 0$.

   (b) The curve given by $y = (2 + x)e^{-x}$, at $(0, 2)$.

   (c) The curve given by $y \sin 2x = x \cos 2y$, at $(\pi/2, \pi/4)$.

8. Suppose that $g(x)$ is given by the following implicit relation $g(x) + x\sin g(x) = x^2$; find $g'(0)$.

9. Show that if $f(x) = (x - a)(x - b)(x - c)$ then

$$\frac{f'(x)}{f(x)} = \frac{1}{x-a} + \frac{1}{x-b} + \frac{1}{x-c}.$$

10. Compute the limit

$$\lim_{x \to 0} x^2 \ln\left(\frac{\sin(x)}{x}\right).$$

11. Let $f(x) = e^{1/\ln(x)}$.

(a) Find the domain in which f can be properly defined.

(b) Find all the vertical and horizontal asymptotes.

(c) Compute the derivative of f.

(d) Compute the second derivative of f.

12. Let $f(x) = \left(1 - \frac{1}{x} + \frac{2}{x^2}\right) e^{1/x}$

 (a) Find the domain in which f can be properly defined.

 (b) Find the zeros, and provide the intervals in which f is constant.

 (c) Find all the vertical and horizontal asymptotes.

 (d) Compute the derivative of f.

 (e) Compute the second derivative of f.

13. Let

$$f(x) = \begin{cases} \frac{x \ln x}{x-1} & \text{if } x > 1, \text{ and } x \neq 1 \\ \alpha & \text{if } x = 1 \end{cases}$$

 (a) find the value of α such that f is continuous in $\mathbb{R}_*^+$.

 (b) Does f' exists for $x > 0$? (be careful when $x = 1$). If it exists, compute $f'(x)$. (**Hint:** You may need to compute by definition the derivative at the problematic points)

 (c) Find where f' is continuous in $\mathbb{R}_*^+$.

14. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is differentiable. Now suppose that f' changes sign at least once. Explain why f can not be one-to-one