1. Find the value of $a$ such that the following functions are continuous in $\mathbb{R}$, and plot them for that value of $a$.

   (a)
$$f(x) = \begin{cases} x^2 & \text{if } x \leq 3 \\ ax & \text{if } x > 3 \end{cases} \tag{1}$$

   (b)
$$f(x) = \begin{cases} 1/x & \text{if } x \leq 1 \\ ax^2 & \text{if } x > 1 \end{cases} \tag{2}$$

   (c)
$$f(x) = \begin{cases} \sin(x)/x & \text{if } x \neq 0 \\ a & \text{if } x = 0 \end{cases} \tag{3}$$

2. Using the Intermediate value theorem, show that there is a root of the equation $\cos(\pi\sqrt{x}) = e^x - 2$ in the interval $(0, 1)$.

3. Using the Squeeze Theorem find the following limits:

   (a)
$$\lim_{x\to 0} x^2 \cos^4\left(\frac{1}{x}\right). \tag{4}$$

   (b)
$$\lim_{x\to 0} x^2 \sin^4\left(\frac{1}{x}\right). \tag{5}$$

   (c)
$$\lim_{x\to\infty} \frac{\sin(x)}{x} \tag{6}$$

   (d)
$$\lim_{x\to\infty} \frac{\cos(x)+4}{x} \tag{7}$$

4. Find all the horizontal and vertical asymptotes of the following functions

   (a)
$$f(x) = \frac{x^2 - x - 6}{x^2 - 2x - 3} \tag{8}$$

   (b)
$$g(x) = \frac{\cos(x)}{\sin(x)} \tag{9}$$

   (c)
$$h(x) = \frac{\cos(x)}{x^2 - 2x - 3} \tag{10}$$

5. Compute the following limits (using the Limit Laws)

   (a)
$$\lim_{x\to 0} \frac{\sqrt{x+1} - 1}{x} \tag{11}$$

(b)

$$\lim_{x\to 1} f(x), \qquad \text{where } f = \begin{cases} (x^2-1)/(x-1) & \text{if } x \neq 1 \\ 7 & \text{if } x = 1 \end{cases} \tag{12}$$

6. Find the tangent line of the following function at $x_0 = 3$

   (a)

$$f(x) = \frac{1}{x} \tag{13}$$

   (b)

$$f(x) = \frac{1}{x^2} \tag{14}$$

   (c)

$$f(x) = \frac{1}{\sqrt{x}} \tag{15}$$

   (d)

$$f(x) = \sin(x) \tag{16}$$

7. Compute the following limits using the comparison Theorems, or show that it does not exists

   (a)

$$\lim_{x\to 1} \frac{1+\sin^2(1/x)}{|x-1|}. \tag{17}$$

   (b)

$$\lim_{x\to \infty} \frac{1+\sin^2(1/x)+\cos^2(x^2)}{|x-1|}. \tag{18}$$

   (c)

$$\lim_{x\to\infty} \frac{P(x)}{Q(x)}; \tag{19}$$

   $P, Q$ polynomials, and the degree of $P$ is greater than the degree of $Q$.

   (d)

$$\lim_{x\to\infty} \frac{P(x)}{Q(x)}; \tag{20}$$

   $P, Q$ polynomials, and the degree of $P$ is less than the degree of $Q$.

8. Find the derivatives of the following functions using the definition, and state the domain of each of the derivatives

   (a)

$$f(x) = 3x - 8 \tag{21}$$

   (b)

$$g(x) = x^2 - x^3 \tag{22}$$

   (c)

$$h(x) = x^{3/2} \tag{23}$$

   (d)

$$f(x) = \sqrt{9-x} \tag{24}$$

(e)

$$f(x) = \frac{1}{\sqrt{x}} \tag{25}$$

9. Let suppose that you have a canon a 2 dimensional world. The amoun of powder is contant and you want to throw a canon ball as fasr as possible. The lonly parameter that you can control is the angle between the cannon and the floor, which we denote by $\theta$.

   You have that the motion is described by

$$x(t) = vt\sin(\theta) \qquad \text{and } y(t) = vt\cos(\theta) - \frac{g}{2}t^2. \tag{26}$$

   (a) Compute the distance the cannon ball will travel in the $x$ direction until it touches the ground ($y = 0$).

   (b) Plot the function that you found and found the $\theta$ for which the distance in $x$ is maximum. (**Hint:** you can use the fact that $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$, and to find the maximum $\theta$, you can take a look at the graph and convince yourself that the maximum will be atteing when the derivative of the distance function is equal to zero.)