

Math 2A
Fall 2016
Midterm 2
11/7/2016

Name (Print): _____

Time Limit: 50 Minutes

Student ID _____

This exam contains 10 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a “theorem” you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	14	
2	15	
3	15	
4	30	
5	14	
6	12	
Total:	100	

Do not write in the table to the right.

1. Compute the derivatives of the following functions:

(a) (7 points)

$$f(x) = x \ln x - x,$$

and compute $f'(1)$.

(b) (7 points)

$$f(x) = \ln(\ln x),$$

and compute $f'(2)$.

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2. (15 points) A cylindrical tank with radius $5[m]$ is being filled with water at a rate of $3[m^3/s]$. How fast is the height of the water increasing? (**Hint:** Make a sketch of the problem, compute the volume of a cylinder and then use implicit differentiation coupled with the chain rule.)

3. (15 points) Using the definition of the derivative (via a limit), compute the derivative of

$$h(x) = \sqrt{x}, \tag{1}$$

state the domain of the derivative, and compute the tangent line at $x = 3$.

4. Compute the derivatives of the following functions:

(a) (10 points)

$$f(x) = (\sin x)^{\ln x}$$

(b) (10 points)

$$f(x) = \tan^{-1} \left(\sqrt{\frac{1 - \cos x}{1 + \cos x}} \right)$$

(c) (10 points)

$$f(x) = (\tan x)^{1/x}$$

5. (14 points) Find the tangent line of the curve given by $e^{2\sin^{-1}(yx)} = \ln(1 + x^2 + y^2)$, at the point in which the curve intersects $y = 0$ such that $x > 0$. (**Hint:** Using implicit differentiation compute dy/dx , and then solve the equation for x when $y = 0$. You will find two roots; use only the one greater than zero)

6. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = xe^x$.

(a) (3 points) Show that f satisfies $f' = e^x + f$.

(b) (3 points) Show that f satisfies $f^{(n)} = e^x + f^{(n-1)}$, for any $n \in \mathbb{N}$, such that $n > 0$

(c) (5 points) Using the questions above find a formula for $f^{(n)}(x)$.

(d) (1 point) Compute $f^{(1000)}(0)$