Math 2A
Fall 2016
Midterm 1
10/17/2016
Time Limit: 50 Minutes

Name (Print): ______________________

Student ID ______________________

This exam contains 11 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a "theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	20	
2	22	
3	15	
4	12	
5	15	
6	16	
Total:	100	

1. Answer the following questions regarding functions

   (a) (5 points) Let $n = 5$, and suppose that

   $$\begin{aligned} f :\ & \mathbb{R} \to \mathbb{R} \\ & x \to f(x) = n!. \end{aligned}$$

   Draw the graph of f.

   (b) (5 points) Let

   $$\begin{aligned} f :\ & \mathbb{R} \to \mathbb{R} \\ & u \to f(u) = u^2, \end{aligned}$$

   and

   $$\begin{aligned} g :\ & \mathbb{R} \to \mathbb{R} \\ & v \to g(v) = v^2. \end{aligned}$$

   Are f and g equal? Explain your answer.

(c) (5 points) Let

$$\begin{aligned} f : \mathbb{R} &\to \mathbb{R} \\ x &\to f(x) = \cos(x), \end{aligned}$$

is f one-to-one? is f onto $\mathbb{R}$? How would you modify the domain and co-domain of f such that the inverse exists? Write down the inverse.

(d) (5 points) Let

$$\begin{aligned} f:\ & \mathbb{R} \to \mathbb{R} \\ & x \to f(x) = 2^x, \end{aligned}$$

is f one-to-one? is f onto $\mathbb{R}$? How would you modify the domain and co-domain of f such that the inverse exists? Write down the inverse.

2. Compute the followign limits if they exist, otherwise, explain why they do not exist.

(a) (3 points)

$$\lim_{x \to 0} \frac{\sqrt{9 - x^2} - 3}{x^2} \tag{1}$$

(b) (3 points)

$$\lim_{x \to 3^-} \frac{x}{x - 3} \tag{2}$$

(c) (3 points)

$$\lim_{x \to 1} \frac{1}{|x - 1|} \tag{3}$$

(d) (3 points)

$$\lim_{x\to 1} \frac{x^2-1}{|x-1|} \tag{4}$$

(e) (5 points)

$$\lim_{x\to 1} \frac{1+\frac{\sin^2(x^2-6)}{cos^2(x^8-3)}+\frac{x^4-1}{x^2-1}}{|x-1|} \tag{5}$$

Hint: try to bound the expression by below, and then use the limit that you computed before and the comparison theorem to find the limit.

(f) (5 points)

$$\lim_{x\to 0} x^2\cos^4\left(\frac{1}{x}\right) \tag{6}$$

Hint: try to bound the expression by below and above, and use the Sandwich theorem.

3. Find all the horizontal and vertical asymptotes of the following functions

(a) (5 points)

$$f(x) = \frac{x^2 - x - 6}{x^2 - 2x - 3} \tag{7}$$

(b) (5 points)

$$g(x) = \frac{1 + \sin^2(1/x) + \cos^2(x^2)}{|x - 1|}. \tag{8}$$

(c) (5 points)

$$h(x) = \frac{\arctan(x)}{x^2 - 2x - 3} \tag{9}$$

4. (12 points) Using the Intermediate value theorem, show that there is a root of the equation $\cos(\pi\sqrt{x}) = e^x - 3$ in the interval $(0, 1)$.

5. Find the values of a and b such that the following functions are continuous in $\mathbb{R}$, and plot them.

   (a) (5 points)

$$f(x) = \begin{cases} x^2 & \text{if } x < 3 \\ b & \text{if } x = 3 \\ ax & \text{if } x > 3 \end{cases} \tag{10}$$

(b) (5 points)

$$f(x) = \begin{cases} 1/x & \text{if } x < 1 \\ b & \text{if } x = 1 \\ (ax)^2 & \text{if } x > 1 \end{cases} \tag{11}$$

(c) (5 points)

$$f(x) = \begin{cases} a\sin(x)/x & \text{if } x \neq 0 \\ b & \text{if } x = 0 \end{cases} \tag{12}$$

6. Find the derivatives of the following functions using the definition, and state the domain of each of the derivatives, acn compute the tangent line at $x = 3$.

   (a) (8 points)

$$h(x) = x^{3/2} \tag{13}$$

(b) (8 points)

$$f(x) = \sqrt{9 - x} \tag{14}$$