Mathematics 234, Lecture 3 (Wilson)

Your Name: ______________________________

Circle your TA's name:

Mark MacLean Adnan Rebei Peter Wiles

Final Exam 12/22/98

Write your answers to the ten problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using π, $\sqrt{3}$, and similar numbers) rather than using decimal approximations.

You may refer to notes you have brought in on up to three sheets of paper (regular notebook or typing size) as announced in class.

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. IF YOU EVALUATE AN INTEGRAL BY SOME MEANS OTHER THAN "BRUTE FORCE," BE SURE TO EXPLAIN WHAT YOU ARE USING!

Problem	Points	Score
1	18	
2	21	
3	20	
4	20	
5	20	
6	20	
7	21	
8	20	
9	19	
10	21	
TOTAL	200	

SCRATCH PAPER

Problem 1 (18 points)
Find <u>all</u> first and second partial derivatives of $f(x, y) = x\,e^y + y + 1$. (If you do not derive all six separately, give a reason why the ones you do derive represent all of them.)

Problem 2 (21 points)
Let $f(x, y, z) = x + x\,\cos(z) - y\,\sin(z) + y$.

(a) In what direction is f increasing most rapidly at $(2, -1, 0)$? (Express the direction as a unit vector.)

(b) Find an equation for the plane through $(2, -1, 0)$ which is tangent to the level surface of f passing through $(2, -1, 0)$.

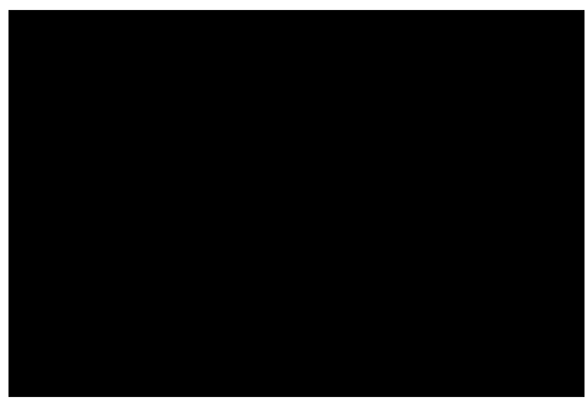
(c) Approximately how much will f change if we move its input (x, y, z) from $(2, -1, 0)$ a distance of 0.2 units toward $(0, 1, 2)$?

Problem 3 (20 points)
Let $\vec{r}(t) = t\,\vec{i} + (1+t^2)\vec{j}$.

(a) Find the value(s) of the parameter t and the point(s) in the plane where $\vec{r}(t)$ and $\vec{r}\,'(t)$ are perpendicular.

(b) Find the value(s) of the parameter t and the point(s) in the plane where $\vec{r}(t)$ and $\vec{r}\,'(t)$ have the same (not opposite) direction.

Problem 4 (20 points)
Use polar or cylindrical coordinates to find the volume of the region which is (a) above the circular disk in the xy-plane $0 \leq x^2 + y^2 \leq 4$ and (b) underneath the bell shaped surface $z = e^{-x^2-y^2}$.



Problem 5 (20 points)
For the curve

$$\vec{r}(t) = 12\cos(t)\,\vec{i} + 12\sin(t)\,\vec{j} + 5t\,\vec{k}$$

find the unit tangent vector $\vec{T}$, the principal unit normal vector $\vec{N}$, the unit binormal vector $\vec{B}$, and the curvature κ, all at the point where $t = \frac{\pi}{4}$.

Problem 6 (20 points)
Let $\vec{r}(t) = t\,\vec{i} + \frac{2}{3}\sqrt{2}\,t^{\frac{3}{2}}\,\vec{j} + \frac{1}{2}\,t^2\,\vec{k}$. Find the arc length along the curve $\vec{r}(t)$ from $t = 0$ to $t = 2$.

Problem 7 (21 points)
Let $f(x, y, z) = 2xy + 3y^2 + z$.

(a) Let $\vec{F}(x, y, z) = \overrightarrow{\nabla} f(x, y, z)$. Write $\vec{F}$ as a combination of $\vec{i}$, $\vec{j}$, and $\vec{k}$.

(b) Evaluate

$$\int_C \vec{F} \cdot d\vec{r}$$

where C is the curve $\vec{r}(t) = 2t\,\vec{i} + (3 - t)\,\vec{j} + t^2\,\vec{k}$, for $0 \leq t \leq 2$.

(c) Evaluate

$$\oint_C \vec{F} \cdot d\vec{r}$$

where C is the boundary of the square with corners $(0, 0)$, $(2, 0)$, $(2, 2)$, and $(0, 2)$.

Problem 8 (20 points)
Find the absolute maximum and minimum values of $f(x, y) = x^2 + xy + y^2 - 6x$ on the rectangular region $0 \leq x \leq 5$, $-3 \leq y \leq 3$.

Problem 9 (19 points)
Find the center of mass of the thin plate bounded by the curves $x = y^2$ and $x = 2y - y^2$, with density $\delta(x, y) = 1 + y$.

Problem 10 (21 points)
Let $\vec{F}(x, y, z) = 3xy\,\vec{i} + 2\sin(z)\,\vec{j} - 3xz^2\,\vec{k}$.

(a) Find $\overrightarrow{curl}\ (\vec{F})$.

(b) Find $div(\vec{F})$.

(c) Find $div(\overrightarrow{curl}\ (\vec{F}))$.