Your Name: ______________________________

Circle your TA's name:

Diego Galindo **James Hunter** **Nansen Petrosyan** **Boian Popunkiov**

Mathematics 234, Fall 2005 **Lecture 2 (Wilson)**

Second Midterm Exam November 17, 2005

Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem. The back page is completely blank and can be used for scratch paper.

Wherever applicable, leave your answers in exact forms (using $\frac{\pi}{3}$, $\sqrt{3}$, $\cos(0.6)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

There is a problem on the back of this sheet: Be sure not to skip over it by accident!

You may refer to notes you have brought on up to two index cards, as announced in class and on the class website.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" (without more details) are not sufficient substantiation...)

Problem	Points	Score
1	14	
2	12	
3	12	
4	12	
5	13	
6	12	
7	12	
8	13	
TOTAL	100	

Problem 1 (14 points)

(a) Let $f(x, y) = x^2 + 2y^2$. Set up, but do not evaluate, an integral to compute the surface area on the graph of f over the region R which is the triangle with vertices $(0, 0)$, $(0, 2)$, and $(1, 0)$.

(b) Set up the following integral with the order of integration reversed, i.e. as an integral with $dy\,dx$ replacing $dx\,dy$. Do not evaluate the integral!

$$\int_1^2 \int_{\sqrt{y-1}}^1 (e^x \cos(y))\, dx\, dy$$

<u>Problem 2</u> (12 points)

Set up an iterated integral to compute $\iiint\limits_R (x + 2y)\, dV$,

where R is the region in space bounded by the cylinder $x^2 + z^2 = 4$, the plane $y = 0$, and the plane $y + z = 2$.
You may choose the order of integration.
You do not have to evaluate this integral, but you can get 5 extra-credit points if you do evaluate it correctly. You will get no extra credit for incorrect evalation of this integral, i.e., this part of the answer will be graded on the correctness of the answer only.

Problem 3 (12 points)

(a) For $f(x, y, z) = x^2 + ye^z$, find the gradient field $\overrightarrow{\nabla} f$.

(b) Let $\overrightarrow{F}(x, y, z)$ be the vector field $2x\vec{\imath} + e^z\vec{\jmath} + ye^z\vec{k}$.

(i) Find $div(\overrightarrow{F})$.

(ii) Find $\overline{curl(\overrightarrow{F})}$.

<u>Problem 4</u>   (12 points)
Find all local maxima, local minima, and saddle points, for $f(x, y) = 6x^2 - 2x^3 + 3y^2 + 6xy$. Be sure to give both the points at which f takes on the values and the values it takes on there.

Problem 5 (13 points)

Evaluate the line integral $\int_C (1-x)ds$ where C is a portion of the circle of radius 2 and center $(0,0)$, traversed from $(2,0)$ to $(0,2)$.

Hint: The curve can be parametrized as $x = 2\cos t$, $y = 2\sin t$, $0 \le t \le \frac{\pi}{2}$.

Problem 6 (12 points)
What are the largest and smallest values that $f(x, y) = xy$ takes on, for (x, y) on the ellipse

$$\frac{x^2}{8} + \frac{y^2}{2} = 1?$$

At what points does f achieve those values?

Problem 7 (12 points)

A thin plate covers the region in the plane bounded by the x-axis, the line $x = 1$, and the curve $y = \sqrt{x}$, where $y \geq 0$.
The density of this plate is given by the function $\delta(x, y) = x + y$.

(a) Use an integral to evaluate the mass of this plate.

(b) Find the moment M_x of the plate about the x-axis.

(c) Find the moment M_y of the plate about the y-axis.

(d) Find the coordinates $(\overline{x}, \overline{y})$ of the center of mass of this plate.

Problem 8 (13 points)

Find the volume of the region that is above the cone $\phi = \frac{\pi}{3}$ and inside the sphere $\rho = 1$. You should set up an integral in spherical coordinates and evaluate it to get a number as your answer.
At the right is a figure cut away so that you can see inside the sphere.