Your Name: ______________________________

Circle your TA's name:

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Mathematics 234, Fall 2004 **Lecture 3 (Wilson)**

Final Exam **December 22, 2004**

Write your answers to the ten problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using $\frac{\pi}{3}$, $\sqrt{3}$, $\cos(0.6)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

There is a problem on the back of this sheet: Be sure not to skip over it by accident!

There is scratch paper at the end of the exam.

You may refer to notes you have brought on index cards or notebook paper, as announced in class and on the class website.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" (without more details) are not sufficient substantiation...)

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	
9	20	
10	20	
TOTAL	200	

Problem 1 (20 points)

For the helical ("corkscrew") motion $\vec{r}(t) = 2\cos(t)\vec{\imath} - 2\sin(t)\vec{\jmath} + 3t\vec{k}$:

(a) Find the velocity $\vec{v}(t)$ as a function of t.

(b) Find the acceleration $\vec{a}(t)$ as a function of t.

(c) Find the unit tangent vector $\vec{T}(t)$.

(d) Find the principal unit normal vector $\vec{N}(t)$.

(e) Find the curvature $\kappa(t)$.

Problem 2 (20 points)

The function $f(x,y) = \dfrac{2xy^2}{x^2 + y^4}$ does not have a limit as $(x,y) \to (0,0)$.

Show that this is true.

Problem 3 (20 points)

Let $f(x, y) = x^2 y + e^{xy} \sin y$.

(a) What is the gradient ∇f as a function of x and y?

(b) At the point $(1, 0)$, in what direction $\vec{u}$ is the directional derivative $D_{\vec{u}} f$ largest? In what direction is the directional derivative smallest?

Express the directions as specific unit vectors. There should not be any x or y in your answers to this part of the problem.

(c) What is the value of the directional derivative at $(1, 0)$, in the direction making the directional derivative largest?

Problem 4 (20 points)
Set up but do not evaluate an iterated integral for the integral of

$$f(x, y, z) = 3x^z - 2z\cos(xy)$$

over the region in space which is:
Inside the cylinder $x^2 + z^2 = 4$ and between the planes $y = 0$ and $x + y = 3$.

Problem 5 (20 points)

For $\int_0^2 \int_0^{\sqrt{4-x^2}} xy\sqrt{x^2+y^2}\, dy\, dx$:

Evaluate the integral by converting to polar coordinates and evaluating the resulting polar integral.

Possibly useful formulas:
$\sin(2\theta) = 2\,\sin(\theta)\cos(\theta)$
$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$

Problem 6 (20 points)
If x, y, and z satisfy $z^3 - xy + yz + y^3 = 2$:

(a) Find $\dfrac{\partial z}{\partial x}$.

(b) Find $\dfrac{\partial z}{\partial y}$.

(c) Find $\dfrac{\partial z}{\partial x}$ at the point $(1, 1, 1)$.

Problem 7 (20 points)
Let $f(x, y, z) = xy + y + z$.
Let C be the curve $\vec{r}(t) = 2t\vec{\imath} + t\vec{\jmath} + (2 - 2t)\vec{k}$ for $0 \leq t \leq 1$.

Evaluate the line integral $\int_C f(x, y, z)\, ds$.

Problem 8 (20 points)
One of the following two vector fields is conservative and the other is not.

$$F_1(x, y, z) = (2x - 3)\vec{\imath} - z\vec{\jmath} + (\cos z)\vec{k}$$

$$F_2(x, y, z) = (e^x \cos y + yz)\vec{\imath} + (xz - e^x \sin y)\vec{\jmath} + (xy + z)\vec{k}$$

(a) Which vector field is conservative? Which one is not conservative?

Show work that leads to your conclusion: You should actually show that one is conservative and show that the other is not, directly. Don't just show one is conservative, or one is not, and use the fact that there is one of each to decide about the other!

(b) For the vector field F that you found to be conservative, evaluate

$$\int_C F(x, y, z)\, ds$$

where C is any path leading from $(0, 0, 0)$ to $(-1, \frac{\pi}{2}, 2)$.

Hint: It is almost surely easier to do this using a potential function!

Problem 9 (20 points)

Evaluate $\oint_C -y^2\,dx + xy\,dy$

around the square in the xy-plane with vertices $(0,0)$, $(1,0)$, $(1,1)$, and $(0,1)$, in that order.

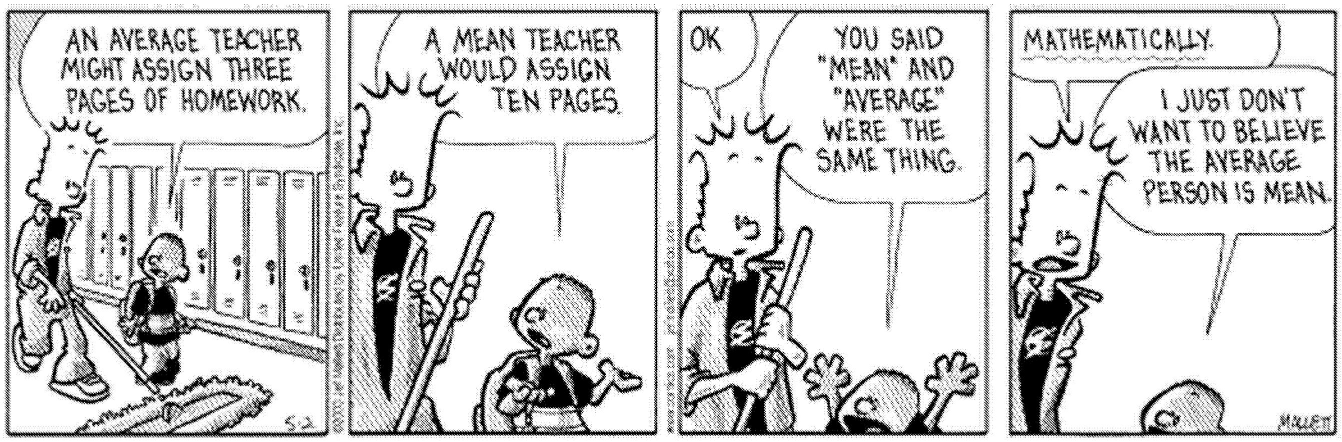
Problem 10 (20 points)

Evaluate $\oint_C \vec{F} \cdot \vec{T}\, ds$ where

$\vec{F}(x, y, z) = y\vec{\imath} + xz\vec{\jmath} + x^2\vec{k}$ and

C is the boundary of the triangle in the plane $x + y + z = 1$ with vertices $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$, traversed counterclockwise as viewed from above.

Hint: Stokes' Theorem could help a lot here...



Scratch Paper