Mathematics 223, Lecture 3 (Wilson) Your Name: _____

Circle your TA's name:

Nicolas Eisen Kellie Evans Jonathan Kochavi

Brian Kruse Huyun Sha

Exam III 5/12/94

Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using π, e, $\sqrt{3}$, $\ln(2)$, and similar numbers) rather than using decimal approximations.

When a problem says to use a particular technique to solve a differential equation, you may receive NO credit unless your answer shows that you can implement that technique.

There is scratch paper at the end of the exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought in on one 4" by 6" index card, as announced in class.

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.

Problem	Points	Score
1	12	
2	12	
3	13	
4	12	
5	14	
6	14	
7	10	
8	13	
TOTAL	100	

Problem 1 (12 points)
Find the solution of

$$\cos(y)\,\frac{dy}{dx} = e^{2x}$$

which satisfies $y(0) = \frac{\pi}{2}$.

You may leave the solution in an implicit form.

Problem 2 (12 points)
Consider the equation $(x + 2y)dx - xdy = 0$:

(a) Show that this equation is NOT exact.

b) USING the fact that $\rho = \frac{1}{x^3}$ is an integrating factor which makes the equation exact, find all solutions to the equation.

Problem 3 (13 points)
Find the solution of

$$x^2 \frac{dy}{dx} + 2xy = 1$$

which satisfies $y(1) = 0$.

Problem 4 (12 points)
Find all solutions to:

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 0$$

Problem 5 (14 points)
Find all solutions to:

$$\frac{d^3y}{dx^3} + 4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} = 0$$

Problem 6 (14 points)
Find all solutions to:

$$\frac{d^2y}{dx^2} + 9y = 6\cos(x)$$

Problem 7 (10 points)
Let $\vec{F} = y^2\vec{i} + 2xy\vec{j}$.
Find a potential function for $\vec{F}$ and USE that to do parts (a) and (b) below:

(a) Find $\int_C \vec{F} \cdot d\vec{r}$ along any parameterized curve C from (1,2) to (3,1).

(b) Find the integral of $\vec{F}$ around the closed curve which is the unit circle traversed once starting at (0,1) and going counterclockwise.

Problem 8 (13 points)
Find all solutions of

$$e^{3x}\ \frac{dy}{dx} + 3e^{3x}\ y = 3x$$

SCRATCH PAPER