Mathematics 223, Lecture 3 (Wilson) Your Name: _____

Circle your TA's name:

Nicolas Eisen Kellie Evans Jonathan Kochavi

Brian Kruse Huyun Sha

Exam I 2/24/94

Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.
Wherever applicable, leave your answers in exact forms (using π, e, $\sqrt{3}$, $\ln(2)$, and similar numbers) rather than using decimal approximations.
You may refer to notes you have brought in on one 4" by 6" index card, as announced in class.

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.

Problem	Points	Score
1	14	
2	10	
3	12	
4	14	
5	18	
6	17	
7	15	
TOTAL	100	

Problem 1 (14 points)

Let $f(x, y, z) = x^2 y - y \sin(z)$.
Let $\vec{v} = 2\vec{i} + \vec{j} - 2\vec{k}$.

(a) Find all first partial derivatives of f.

(b) Find $\frac{\partial^3 f}{\partial z \partial y \partial z}$. Be sure to indicate the order in which you differentiate, or else state carefully a theorem which makes it irrelevant.

(c) Find the directional derivative of f in the direction of the vector $\vec{v}$.

Problem 2 (10 points)

Let $f(x, y) = \sin(2x + y)$.

Starting from $f(0, \frac{\pi}{2}) = 0$, use partial or directional derivatives to estimate $f(0.1, \frac{\pi}{2} - .2)$.

YOU choose whether to use a technique we have covered which used partial derivatives or one which used directional derivatives.
You do NOT need to give an error bound!
You DO need to use derivatives, and show how you have done so: Just calculating the value of $f(0.1, \frac{\pi}{2} - .2)$ using a calculator is NOT sufficient!

Problem 3 (12 points)

Let $f(x, y, z) = xyz - x^2 - 2y^2$. Let $P_0 = (1, -1, 2)$.

(a) (4 points) Find $\overrightarrow{\nabla} f$.

(b) (4 points) Find $\overrightarrow{\nabla} f \mid_{P_0}$.

(c) (4 points) Find a <u>unit</u> vector in the direction in which f is increasing most rapidly at P_0.

(d) (4 points) Find the derivative of f in the direction of the vector from (c).

Problem 4 (14 points)

Let $f(x) = x^2 - 2y^2 + 3z^2$.

Find both the tangent plane and the normal line to the level surface $f(x, y, z) = 5$ at the point $P_0 = (2, 1, -1)$.

Problem 5 (18 points)

Let $f(x) = 2\,x^3 + 3\,x^2 - 12\,x + y^3 - 3\,y^2$.

Find all critical points of $f(x)$. For each critical point tell whether it is a local or absolute maximum or a local or absolute minimum for $f(x)$.

Problem 6 (17 points)

A rectangular box is to built with all six sides enclosed. The box has a volume of 2 cubic feet. The vertical sides are made of a material which costs \$1.00 a square foot. The top is made of material which costs \$2.00 a square foot. The bottom is made of material which costs \$1.50 a square foot.

What should be the dimensions of the box so as to minimize the cost of building it?

Problem 7 (15 points)

Use Taylor's formula for $f(x, y)$ to find a quadratic polynomial approximating $e^y sin(x)$ near the origin.