Mathematics 223, Lecture 4 (Wilson) Your Name: ____________________

Circle your TA's name:

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Exam I 10/10/95

Write your answers to the seven problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.
Wherever applicable, leave your answers in exact forms (using π, e, $\sqrt{3}$, $\ln(2)$, and similar numbers) rather than using decimal approximations.
You may refer to notes you have brought in on one sheet of paper, as announced in class.

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.

Problem	Points	Score
1	12	
2	19	
3	14	
4	17	
5	12	
6	10	
7	16	
TOTAL	100	

Problem 1 (12 points)

Let $f(x, y, z) = ye^x - yz^2 + xy$.

Find both the tangent plane and the normal line to the level surface $f(x, y, z) = -6$ at the point $P_0 = (0, 2, -2)$.

Problem 2 (19 points)

Let $f(x, y, z) = x^2y + y^2z - 2xz$. Let $P_0 = (1, 2, -2)$.

(a) Find $\overrightarrow{\nabla f}$.

(b) Find $\overrightarrow{\nabla f}\,|_{P_0}$.

(c) Find a <u>unit</u> vector in the direction in which f is increasing most rapidly at P_0.

(d) Find the derivative of f in the direction of the vector from (c).

(e) Find a non-zero vector $\vec{v}$ such that the directional derivative of f at P_0 in the direction of $\vec{v}$ is zero.

Problem 3 (14 points)

Let $f(x, y) = x^3 + y^3 - 3x^2 + 3y^2 + 4$.

Find all local maxima, local minima, and saddle points of f. For each local maximum or minimum tell whether it is also an absolute maximum or minimum.

Problem 4 (17 points)

Let $f(x,y,z) = x^3 + 3x^2y + 2x\sin z$.
Let $\vec{v} = \vec{i} - 2\vec{j} + 2\vec{k}$.
Let $P_0 = (2,-2,0)$.

(a) Find all first partial derivatives of f.

(b) Find $\frac{\partial^3 f}{\partial z \partial x \partial z}$. Be sure to show clearly that you are differentiating in the order prescribed, or else state carefully a theorem which makes it irrelevant.

(c) Find the directional derivative of f in the direction of the vector $\vec{v}$. (Find it in general, not at any particular point.)

(d) Find the directional derivative of f in the direction of the vector $\vec{v}$ at the point P_0.

Problem 5 (12 points)

Let $f(x,y) = e^{xy} + \cos(\pi y^2)$, $x(u,v) = v \ \ln u$, and $y(u,v) = u^2 + v$.

(a) Find $\frac{\partial f}{\partial u}$ and $\frac{\partial f}{\partial v}$. You may leave the variables x, y, u, v in your answer.

(b) Find $\frac{\partial f}{\partial u}$ at the point $(u,v) = (1,2)$. This should turn out to be a number, with no x, y, u, v remaining.

Problem 6 (10 points)

Let $f(x, y, z) = x^2 + 4xy - 2xz$.

Use the "standard linear approximation" and the fact that $f(1, 1, 1) = 3$ to find an approximate value of $f(0.9, 1.2, 1)$.

You do not have to give an error estimate.
You do have to use calculus: Just using a calculator to compute what value f has at that point will get no credit.

Problem 7 (16 points)

Let $f(x, y) = xy - x^2 - y^2 + 3$. Find the absolute maxima and minima of f on the closed square region in the plane bounded by the four lines $x = 1$, $x = -1$, $y = \frac{1}{2}$, and $y = -1$.