Your Name: ______________________________

Circle your discussion instructor's name:

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Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using π, $\sqrt{3}$, $\cos^{-1}(0.6)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

You may refer to notes you have brought in on a sheet of paper, as announced in class.

On the other side of this sheet there is a collection of facts and formulas. In a problem where you are asked to evaluate an integral, you may directly use these formulas. If the function you are asked to integrate does not directly fit these formulas, you must show all work needed to do the integral using these formulas together with substitution and the techniques studied in chapter seven of the text. Do not simply quote some other formula and "plug in" the function.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" are not sufficient substantiation...)

Problem	Points	Score
1	15	
2	10	
3	15	
4	15	
5	15	
6	5	
7	12	
8	13	
TOTAL	100	

Some formulas, identities, limits, and numeric values you might find useful:

Values of trig functions:

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	—

Trig facts:

1. $\tan\theta = \frac{\sin\theta}{\cos\theta}$
2. $\sec\theta = \frac{1}{\cos\theta}$
3. $\sin^2\theta + \cos^2\theta = 1$
4. $\sec^2\theta = \tan^2\theta + 1$
5. $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$
6. $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$
7. $\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$
8. $\sin^2 x = \frac{1}{2}(1-\cos 2x)$
9. $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

Derivative formulas:

1. $\frac{d}{dx}\tan x = \sec^2 x$
2. $\frac{d}{dx}\sec x = \sec x\,\tan x$
3. $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$
4. $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$
5. $\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$
6. $\frac{d}{dx}\ln x = \frac{1}{x}$
7. $\frac{d}{dx}e^x = e^x$

Integral formulas: (We assume you know, and you are certainly allowed to use, basic formulas for integrals of functions such as x^n, e^x, $\sin x$, $\cos x$, etc., and how to use substitution to extend these.)

1. $\int \frac{1}{u}\,du = \ln|u| + C$
2. $\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C$
3. $\int \frac{du}{1+u^2} = \tan^{-1} u + C$
4. $\int \sec(u)\,du = \ln|\sec(u)+\tan(u)| + C$
5. $\int u\,dv = uv - \int v\,du$

Algebra formulas:

1. $a^{x+y} = a^x\,a^y$
2. $a^b = e^{b\ln a}$
3. $\ln(xy) = \ln(x) + \ln(y)$

Some commonly encountered limits:

1. $\lim\limits_{n\to\infty} \dfrac{\ln n}{n} = 0$
2. $\lim\limits_{n\to\infty} \sqrt[n]{n} = 1$
3. $\lim\limits_{n\to\infty} x^{\frac{1}{n}} = 1 \quad (x > 0)$
4. $\lim\limits_{n\to\infty} x^n = 0 \quad (|x| < 1)$
5. $\lim\limits_{n\to\infty} \left(1 + \dfrac{x}{n}\right)^n = e^x \quad (\text{any } x)$
6. $\lim\limits_{n\to\infty} \dfrac{x^n}{n!} = 0 \quad (\text{any } x)$

Problem 1 (15 points)

Evaluate the integrals:

(a) $$\int \frac{\sqrt{4x^2-9}}{2x}\,dx$$

(b) $$\int \cos^2(x)\sin^3(x)\,dx$$

(c) $$\int x\cos(2x)\,dx$$

Problem 2 (10 points)
Evaluate the integrals:

(a) $$\int_0^3 \frac{dx}{(x-2)^{\frac{2}{3}}}$$

(b) $$\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^2 x}\, dx$$

Problem 3 (15 points)

For each of these sequences: (i) Tell whether it converges or not. (ii) If it converges, tell what its limit is. You should be sure to give reasons, but you do not need the precision of ϵs and the formal definition of the limit of a sequence.

(a) $$a_n = \sin\left(\frac{\pi}{3} + \frac{1}{n}\right)$$

(b) $$a_n = \sin\left(\frac{\pi}{3} + n\pi\right)$$

(c) $$a_n = \left(\frac{n-1}{n}\right)^n$$

Problem 4 (15 points)

For each of the following series: (i) Tell whether it converges. (ii) If it converges, tell what it converges to. Be sure to give reasons for your answers!

(a) $$\sum_{n=0}^{\infty}\left(\frac{4}{3^n}-\frac{1}{2^n}\right)$$

(b) $$\sum_{n=0}^{\infty}\left(\left(\frac{4}{3}\right)^n-\frac{1}{2^n}\right)$$

(c) $$\sum_{n=1}^{\infty}\frac{n^2-4}{n}$$

Problem 5 (15 points)

For each of the following series: (i) Tell whether it converges absolutely, converges conditionally, or diverges. Be sure to give reasons for your answers!

(a) $$\sum_{n=1}^{\infty} \frac{\cos(n)}{n^{\frac{3}{2}}}$$

(b) $$\sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln(n^4)}$$

(c) $$\sum_{n=1}^{\infty} (-1)^n \frac{n}{3n+5}$$

Problem 6 (5 points)

For the power series

$$\sum_{n=0}^{\infty} \frac{(x+3)^n}{5^n}$$

Find the radius of convergence, and also find the interval of convergence. Be sure to show your reasoning.

Problem 7 (12 points)

(a) Find the Maclaurin series for $f(x) = \ln(1 - x)$.

You don't have to write a precise formula for the n^{th} term, if you show enough terms that there is an obvious pattern as to what appears in each place. (The grader determines what is "obvious"!) You can receive up to 3 bonus points for giving a precise formula for the n^{th} term, however.

You may be able to find this series using some tools other than direct computation using the formula for the terms of Maclaurin's series. That is fine, but if you do so be sure to tell exactly what you are doing.

(b) For what values of x does the series converge absolutely?

Problem 8 (13 points)
Suppose we use the terms of the Maclaurin series for $f(x) = e^{3x}$ through the x^3 term to compute an approximation to $e^{0.3}$. Give a bound for how far the approximation might differ from the true value. Use a calculus result to produce your bound: No credit will be given for using a calculator to give the difference between these numbers.
You may wish to use the fact that $e < 3$. Be careful in choosing a method not to apply an alternating-series result unless the series is truly alternating!