Mathematics 222, Lecture 2 (Wilson)

Your Name: ____________________________________

Circle your TA's name:

Simon Davies **Manuel Fernandez**

Thomas Hangelbroek **Cristian Tiu**

Exam II 4/8/98

Write your answers to the six problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using π, $\sqrt{3}$, and similar numbers) rather than using decimal approximations. There is scratch paper on page 2. If you need more scratch paper, please ask for it.

You may refer to notes you have brought in on one sheet of paper (regular notebook or typing size) as announced in class.

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.

Problem	Points	Score
1	18	
2	17	
3	15	
4	16	
5	20	
6	14	
TOTAL	100	

SCRATCH PAPER

Problem 1 (18 points)
A particle moves along the curve $x = 3y^2$ from $(3, 1)$ to $(18, 2)$ as time t goes from 0 to 1.

(a) Give a parametric representation of the position of the particle at time t.

(b) Set up, but do not evaluate, an integral to compute the distance traveled by the particle. Your integral should be a definite integral including the appropriate end points.

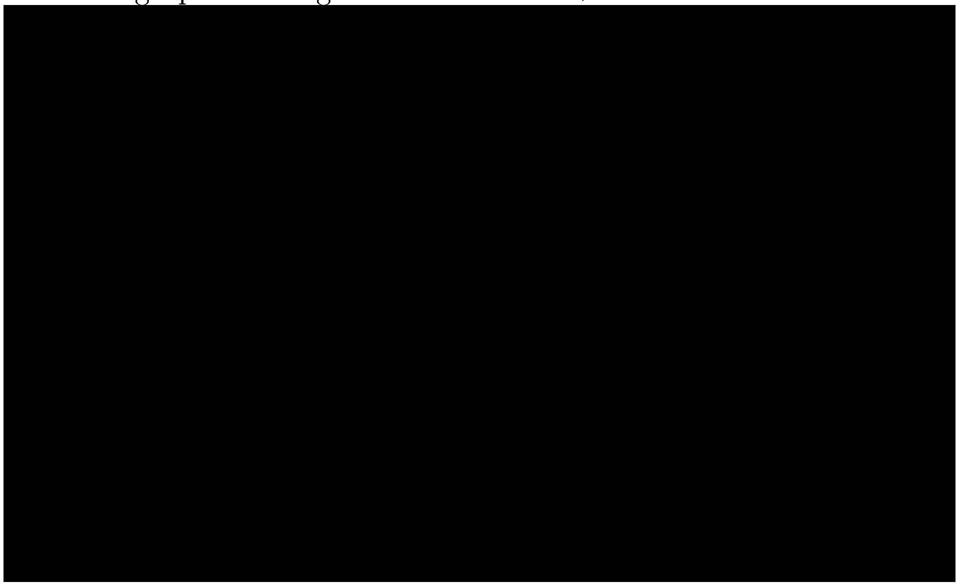
Problem 2 (17 points)
Consider the curve in polar coordinates $r = \cos\theta + \sin\theta$.

(a) Find an equation for the tangent line to this curve at the point where $\theta = \frac{3\pi}{4}$.

(b) Locate the points on the curve where the tangent line is horizontal and the points where it is vertical. Give polar coordinates for each such point.

Problem 3 (15 points)

Here is a graph showing the curves $r = 1 + \cos\theta$ and $r = 3\cos\theta$:



Find the area which is inside <u>both</u> curves. (That means each point is inside both curves, i.e. the intersection of the two regions, not the points which are inside at least one!)

Problem 4 (16 points)

(a) Find an equation for the hyperbola with foci $(0, \pm 5)$ and asymptotes $y = \pm \frac{4}{3}x$.

(b) What kind of conic section is represented by $9x^2 + 4y^2 = 36$? Where are its foci?

Problem 5 (20 points)

(a) Find the sum of this series:

$$\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n+1)}$$

(b) For each of these series, does the series converge? If so give the sum of the series, and if not tell how you know that it does not?

$$\sum_{n=1}^{\infty} \frac{0.001}{(0.9)^n}$$

$$10 + 9 + 8.1 + 7.29 + 6.561 + \ldots \text{ (each term is 0.9 times the previous one)}$$

(c) For each of these series, tell whether it converges and how you know: You do not have to tell what the sum is if it converges.

$$\sum_{n=1}^{\infty} \frac{1}{n-0.1}$$

$$\sum_{n=1}^{\infty} \frac{10^n}{n!}$$

Problem 6 (14 points)

(a) Does the series

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{n}$$

converge? Does it converge absolutely? If it converges (whether absolutely or not), how many terms would you need to add up to be within 0.0001 of the complete sum?

(b) Does the series

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{n^2}$$

converge? Does it converge absolutely? If it converges (whether absolutely or not), how many terms would you need to add up to be within 0.0001 of the complete sum?