Mathematics 222, Lecture 2 (Wilson)   Your Name: _____

Circle your TA's name:

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Exam III   5/9/95

- Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure to tell where to look for the answer, and to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.

- You may use your calculator in doing these problems. If you do some of the work using a calculator, however, be sure to tell precisely what you asked the calculator to do. On all answers be sure to show your work.

- Unsupported answers, even if they give the correct final answer, may receive little or no credit.

- Wherever possible, (even in calculator-assisted answers!) leave your answers in exact forms (using π, e, $\sqrt{3}$, $\ln(2)$, and similar numbers) rather than using decimal approximations.

- There is scratch paper attached. If you need more scratch paper, please ask for it.

- You may refer to notes you have brought in on one sheet of paper.

- Be sure to use some notation that makes clear which things you write are vectors and which are numbers.

Problem	Points	Score
1	12	
2	12	
3	12	
4	14	
5	14	
6	14	
7	10	
8	12	
TOTAL	100	

Problem 1   (12 points)
Find the area of the region which is inside both $r = 2$ and $r = 4\sin\theta$.

Problem 2   (12 points)
For $\vec{\mathbf{A}} = 2\vec{\mathbf{i}} + 10\vec{\mathbf{j}} - 11\vec{\mathbf{k}}$ and $\vec{\mathbf{B}} = -2\vec{\mathbf{i}} + 2\vec{\mathbf{j}} - \vec{\mathbf{k}}$, find:

(i) $|\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}|$

(ii) $|\vec{\mathbf{A}}|$ and $|\vec{\mathbf{B}}|$

(iii) the cosine of the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, <u>and</u> the angle (in radians) itself

(iv)   the projection of $\vec{\mathbf{B}}$ onto $\vec{\mathbf{A}}$

Problem 3   (12 points)

Let P, Q, and R be the points $(1, 2, -1)$, $(3, 5, 0)$, and $(0, 3, 0)$ respectively.

(i)   Find a vector $\vec{\mathbf{N}}$ which is perpendicular to the plane containing P, Q, and R.

(ii)   Find an equation for the plane containing P, Q, and R.

(iii)   Find the area of the triangle with vertices P, Q, and R.

Problem 4   (14 points)
Consider the two planes $2x + 3y + z = 4$ and $-x + y + z = -1$.

(i)   What is the angle between these planes?

(ii)   Find parametric equations for the line of intersection of these planes.

Problem 5   (14 points)

An object moves in space such that its acceleration vector is $\vec{\mathbf{a}}(t) = 18t\,\vec{\mathbf{i}} + 12t^2\,\vec{\mathbf{j}} + 2\vec{\mathbf{k}}$ at time t.
When $t = 1$ it is at the point $(5, -4, 2)$, and at that same time its velocity is $\vec{\mathbf{v}}(1) = 9\vec{\mathbf{i}} - \vec{\mathbf{j}} + 2\vec{\mathbf{k}}$.
Find the position vector $\vec{\mathbf{r}}(t)$ for this object as a function of time t.

Problem 6   (14 points)
A projectile will be launched with initial speed 800 meters/second. It has to land on a point 30 kilometers away, at the same height as the launch point. What launch angles (there are two of them!) will accomplish this? (For meters and seconds, $g = 9.8$: One kilometer is 1000 meters.)

Problem 7   (10 points)

A "rhombus" is a figure with four sides, opposite sides parallel, and all sides the same length. (Sort of a 'squished square'.) (See picture below for one example where two of the sides are shown as vectors.) The diagonals of a rhombus are the two lines which connect opposite corners. Use vector arithmetic to show that the diagonals of a rhombus meet at a right angle.

Hint: Give names to the vectors. Find new vectors in the directions of the diagonals, written as some combinations of the first two. Now find the angle between these two vectors along the diagonals.
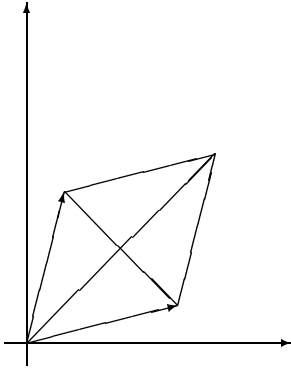
Problem 8   (12 points)
Consider the curve $y = \cos(x)$. At the point $(\frac{\pi}{3}, \frac{1}{2})$, find:

(i)   Two vectors of unit length which are tangent to the curve.

(ii)   Two vectors of unit length which are normal to the curve.

SCRATCH PAPER