Your Name: _____

Circle your TA's name:

Adam Berliner John Bowman Chris Holden Eugene Tsai Dan Turetsky

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Mathematics 222, Spring 2005 Lecture 4 (Wilson)

First Midterm Exam February 15, 2005

Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using $\frac{\pi}{3}$, $\sqrt{3}$, $\cos(0.6)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

You may refer to notes you have brought on an index card, as announced in class and on the class website.
There are also some formulas given on the other side of this sheet.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" (without more details) are not sufficient substantiation...)

Problem	Points	Score
1	12	
2	13	
3	13	
4	13	
5	13	
6	12	
7	12	
8	12	
TOTAL	100	

Some formulas, identities, and numeric values you might find useful:

Values of trig functions:

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	—

Trig facts:

1. $\tan\theta = \frac{\sin\theta}{\cos\theta}$
2. $\sec\theta = \frac{1}{\cos\theta}$
3. $\sin^2\theta + \cos^2\theta = 1$
4. $\sec^2\theta = \tan^2\theta + 1$
5. $\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$
6. $\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$
7. $\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$
8. $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
9. $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

Derivative formulas:

1. $\frac{d}{dx}\tan x = \sec^2 x$
2. $\frac{d}{dx}\sec x = \sec x\,\tan x$
3. $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$
4. $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$
5. $\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$
6. $\frac{d}{dx}\ln x = \frac{1}{x}$
7. $\frac{d}{dx}e^x = e^x$

Integral formulas:

1. $\int u^n\,du = \frac{1}{n+1}\,u^{n+1} + C$, if $n \neq -1$
2. $\int \frac{1}{u}\,du = \ln|u| + C$
3. $\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C$
4. $\int \frac{du}{1+u^2} = \tan^{-1} u + C$
5. $\int \sec(u)\,du = \ln|\sec(u)+\tan(u)| + C$
6. $\int u\,dv = uv - \int v\,du$

Algebra formulas:

1. $\ln(xy) = \ln(x) + \ln(y)$
2. $a^{x+y} = a^x\,a^y$
3. $a^b = e^{b\ln a}$

Problem 1 (12 points)

Evaluate the integral: $\int \sin^2(x)\cos^2(x)\,dx.$

Problem 2 (13 points)

Evaluate the integral: $\displaystyle\int_3^{3\sqrt{2}} \frac{\sqrt{x^2-9}}{x}\,dx$

Problem 3 (13 points)

Evaluate the integral: $\int x^2 \sin(x)\, dx$

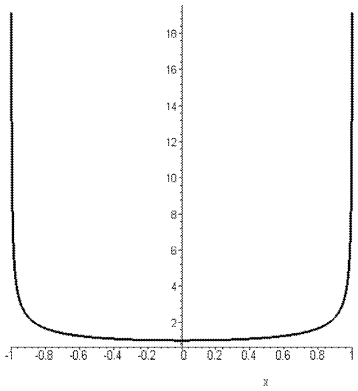
Problem 4 (13 points)

Evaluate the integral: $$\int \frac{4x^2 + 5x + 3}{(x+1)(x^2+x+1)}\,dx$$

Problem 5 (13 points)

At the right is a graph of $\dfrac{1}{\sqrt{1-x^2}}$.

Evaluate the integral $\displaystyle\int_{-1}^{1} \frac{dx}{\sqrt{1-x^2}}$.



Problem 6 (12 points)

(a) Evaluate the limit $\lim_{x\to 0} \dfrac{e^x - e^{-x}}{4\,\sin(x)}$.

(b) Evaluate the limit $\lim_{x\to\infty} x^{\left(2/x^2\right)}$.

Problem 7 (12 points)

(a) Consider the sequence $a_n = 3 - \frac{2}{n}$.

Give an argument to show that $\{a_n\}$ has a limit. You should either justify this using carefully the definition of the limit of a sequence or by appropriate use of a theorem about sequences from the book.

(b) The first several terms of a different sequence $\{a_n\}$ are $1, \frac{2}{2^2-1^2}, \frac{3}{3^2-2^2}, \frac{4}{4^2-3^2}, \ldots$.

(i) Find a formula giving a_n as a formula involving n.

(ii) This sequence does converge. What is its limit? Show all of your work.

Problem 8 (12 points)

For each series, tell whether it converges or diverges.
If the series converges, tell what its sum is.
Be sure to show your work!

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{n-1}{n+1}.$

(b) $\displaystyle\sum_{n=2}^{\infty} \left(\frac{2}{3}\right)^n$