Mathematics 222, Lecture 1 (Wilson)

Your Name: _____

Circle your TA's name:

Gautam Bharali Jeremy Hoffmann Prabu Ravindran

Jamie Sutherland Nat Thiem

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Write your answers to the seven problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.
Wherever applicable, leave your answers in exact forms (using π, $\sqrt{3}$, and similar numbers) rather than using decimal approximations. If you need scratch paper, please ask for it.
You may refer to notes you have brought in on two index cards, as announced in class. You may use a calculator, but remember:
BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.

Problem	Points	Score
1	14	
2	14	
3	14	
4	14	
5	15	
6	15	
7	14	
TOTAL	100	

Problem 1 (14 points)

(a) Find an equation (in x-y-coordinates) for the ellipse with foci at $(0, \pm 6)$ which passes through $(0, 5)$.

(b) A curve has the equation in polar coordinates $r = \frac{6}{3+2\cos(\theta)}$.

(i) What kind of conic section (circle, parabola, hyperbola, ellipse) is it?

(ii) What is its eccentricity?

Problem 2 (14 points)

Consider the curve $y = \cos^2(t)$, $x = 4\sin(t)$:

(a) Find an equation for the tangent line to this curve at the point $(4, 0)$.

(b) Is the curve concave upward or downward at $(4, 0)$? How can you tell?

Problem 3 (14 points)
A particle moves around a circle, its position given by parametric equations $x = 4\cos(t)$ and $y = 4\sin(t)$ at time t. How far does it move along its path between $t = 1$ and $t = 2$?
(Set up and evaluate an integral to compute your answer. There is an easy way to do this problem without calculus: You could use it to check your answer, but it will not be given credit as a solution.)

Problem 4 (14 points)
How many terms of the series

$$\sum_{n=1}^{\infty} \frac{(-2)^{n+1}}{n^2}$$

would be needed to compute a sum which was within 0.01 of the sum of the whole series?
Be sure to explain how you got your answer!

Problem 5 (15 points)
For each series:
(i) Does it converge?
(ii) If it converges, find its sum. If it diverges, tell how you know that.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3}{2n-2}$

(b) $\displaystyle\sum_{n=2}^{\infty} \left(\frac{2}{3}\right)^n$

(c) $\displaystyle\sum_{n=1}^{\infty} 2 + \sin(n)$

Problem 6 (15 points)

For each series, tell if it is absolutely convergent, conditionally convergent, or divergent. Be sure to give reasons for your answers!

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{(-5)^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \, \frac{n^2}{n!}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \, \frac{n}{n^2+1}$

Problem 7 (14 points)

Find the area of the region which is inside $r = 3\sin(\theta)$ but outside $r = 1 + \sin(\theta)$.