Your Name: ____________________________________

Circle your TA's name:

Gautam Bharali **Jeremy Hoffmann** **Prabu Ravindran**

Jamie Sutherland **Nat Thiem**

Exam I March 2, 2000

THERE IS A PROBLEM ON THE BACK OF THIS SHEET!

Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using π, $\sqrt{3}$, and similar numbers) rather than using decimal approximations.

There is scratch paper at the end of the exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought in on one index card, as announced in class. You may use a calculator, but remember:

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" is not sufficient substantiation...)

Problem	Points	Score
1	14	
2	14	
3	13	
4	12	
5	12	
6	10	
7	12	
8	13	
TOTAL	100	

Problem 1 (11 points)

Integrate:

(a) $$\int \frac{3x^2 + 4x + 3}{(x-1)(x^2 + 2x + 2)}\, dx$$

(b) $$\int_0^{\frac{\pi}{2}} e^x \cos(2x)\, dx$$

Problem 2 (14 points)

Integrate:

(a) $$\int \frac{\sqrt{x^2+9}}{x^4}\,dx$$

(b) $$\int_2^{\infty} x\,e^{-2x}\,dx$$

The table shows some values of a function $f(x)$:

x	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1
$f(x)$	-3	-3	-1	3	9

For this function f:

(a) Use the Trapezoidal Rule with $n = 4$ to approximate $\int_{-1}^{1} f(x)\,dx$.

(b) Use Simpson's Rule with $n = 4$ to approximate $\int_{-1}^{1} f(x)\,dx$.

(c) The function $f(x) = 4x^2 + 6x - 1$ takes on the values in the table above. Use the Fundamental Theorem of Calculus to evaluate $\int_{-1}^{1} f(x)\,dx$ exactly for this function. Compare the result to your answers for (a) and (b): Give reasons if an approximation turned out higher instead of lower than the exact result, and also discuss why it happened if an approximation was exactly right.

Problem 4 (12 points)

For the differential equation $y'' + 6y' + 25y = 0$:

(a) Find the general solution, i.e. an expression describing all solutions of that equation.

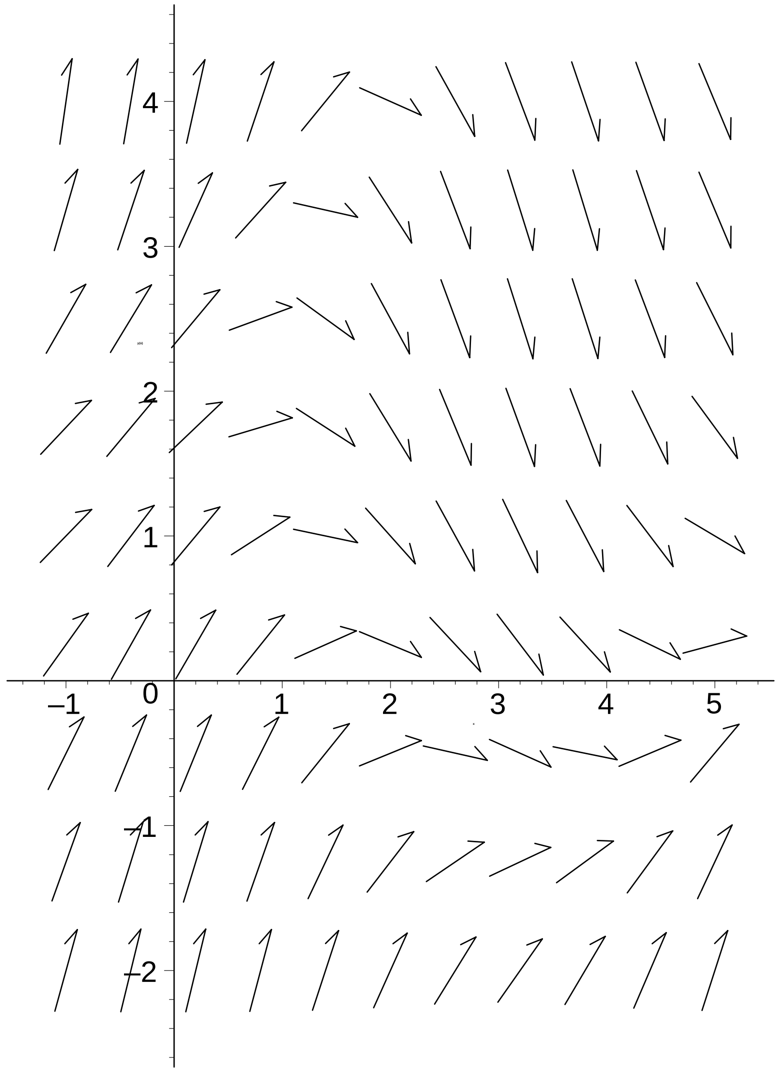
(b) Find the solution to the equation which also satisfies the initial conditions $y(0) = 2$ and $y'(0) = -10$.

Problem 5 (12 points)

Solve the initial value problem: $\frac{dy}{dx} = \frac{\sin(x)}{6y^2-3}$ and $y(0) = 2$.

(It may not be practical to solve completely for $y(x)$. An answer which defines $y(x)$ implicitly is acceptable. Of course that answer should no longer involve derivatives of $y(x)$, e.g. the original equation is not itself an acceptable answer. As an example, although this is not the correct answer, an answer of the form $y^2 + \sin(y) = \cos(x) + 7$ would be in an acceptable form.)

Problem 6 (10 points)



The picture above shows the direction field for a first order differential equation.

(a) Sketch on the picture the graphs of two solutions of the equation, one satisfying $y(0) = -1$ and the other $y(0) = 2.5$. Be sure it is clear which of your solutions goes with which initial condition.

(b) Approximately what will $y(3)$ be, for each of those initial conditions? (Your answer should be two numbers. Be sure you indicate which number goes with which initial condition.)

Problem 7 (12 points)
For the differential equation $(x^2+3)y' + 2xy = \sin(x)$:

(a) Find the general solution. (Remark: This equation can be put in the standard form we have used for a first order linear equation. If you use that technique to solve it you may note an interesting effect which indicates that there was an easier way...)

(b) Find the solution to this equation which satisfies $y(\pi) = 3/(\pi^2+3)$.

Problem 8 (13 points)
Find all solutions to the differential equation

$$y'' + y' - 6y = 28e^{4x}.$$

SCRATCH PAPER