Mathematics 222, Lecture 2 (Wilson) **Your Name:** _____

Final Exam 12/18/93

Circle your TA's name:

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Daode Huang **Jonathan Kochavi** **Chang Ock Lee**

Write your answers to the twelve problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using π, e, $\sqrt{3}$, $\ln(2)$, and similar numbers) rather than using decimal approximations. If you draw a sketch and derive a numerical answer from it, rather than from an analytical process, the sketch must include labels and units which justify the numerical value.

There is scratch paper at the end of the exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought in on 4" by 6" index cards, as announced in class.

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.

Problem	Points	Score
1	18	
2	16	
3	16	
4	16	
5	17	
6	16	
7	14	
8	16	
9	20	
10	16	
11	15	
12	20	
TOTAL	200	

Problem 1 (18 points)

(a) Evaluate the integral

$$\int \cos^6(x) \sin^3(x) dx$$

(b) Evaluate the integral

$$\int \cos^2(x) \sin^2(x) dx$$

(c) Evaluate the integral

$$\int e^{-x} \cos(x) dx$$

Problem 2 (16 points)

(a) Evaluate the integral

$$\int_{\frac{\sqrt{3}}{3}}^{1} \frac{2\ dx}{x\sqrt{4x^2-1}}$$

(b) Evaluate the integral

$$\int_{1}^{\infty} \frac{2\ x\ dx}{(x^2+1)^3}$$

Problem 3 (16 points)

(a) For each of the following series tell whether it converges *absolutely, conditionally,* or *not at all.*

$$\sum_{n=1}^{\infty} \frac{(-1)^n(1+n)}{n^2}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n(1+n)}{n^3}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n(1+n)}{n}$$

(b) For the series

$$\sum_{n=1}^{\infty} \frac{n\ x^n}{5^n}$$

find the interval of convergence. You do *NOT* need to determine convergence at the ends of that interval.

Problem 4 (16 points)

Give the Taylor series for $ln(x)$ at $a = 1$. You should give explicitly the terms through the one with a fourth power, and also give a description either in words or a formula for the n^{th} degree term. That description should say what the terms are for *this* series, not for a Taylor series in general!

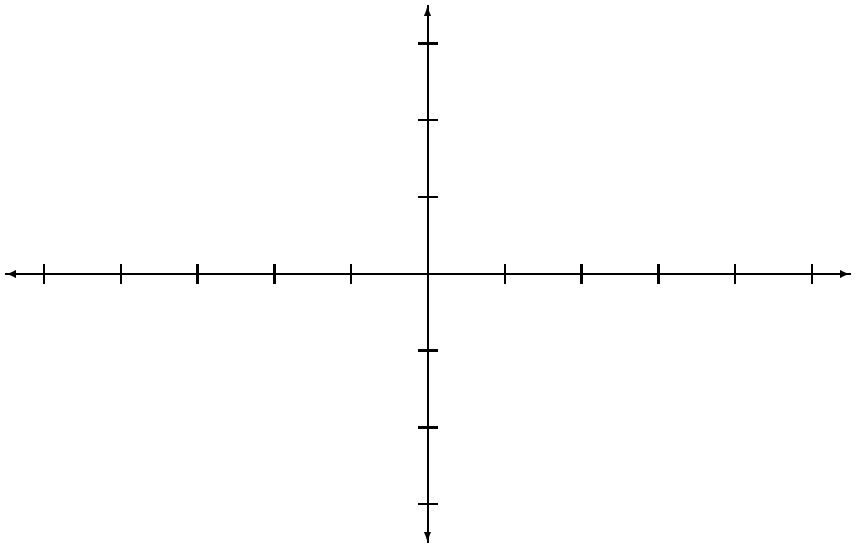
Problem 5 (17 points)

The terms through the one with x^4 of the Maclaurin series for $f(x) = \sin(x) + \cos(x)$ are used to compute an approximate value for $f(0.1)$. Give an estimate of the difference between this approximation and the actual value of $\sin(0.1) + \cos(0.1)$.

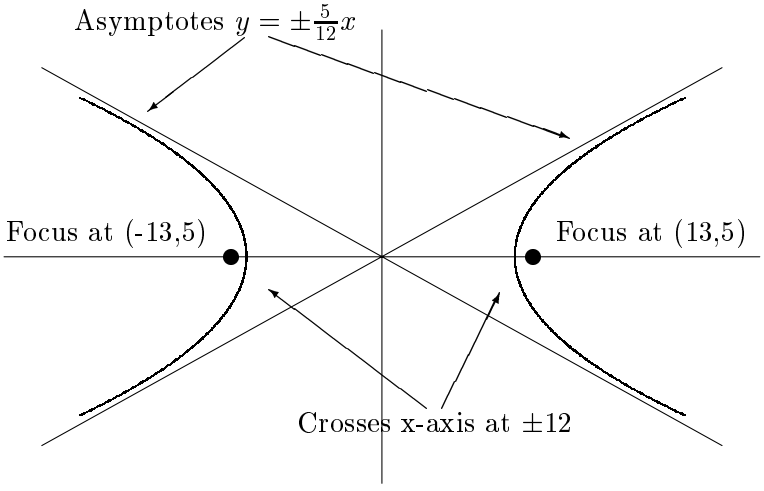
Do NOT simply give the difference between these numbers as approximated by your calculator: Your answer must be based on Taylor's theorem with remainder.

Problem 6 (16 points)

(a) Sketch a graph on the axes below of the curve described by $\frac{x^2}{9} + \frac{y^2}{25} = 1$. Be sure to put numbers on the axes showing the scale, and to identify the vertices and foci of the figure.



(b) Give an equation for the conic section shown below. What is its eccentricity?



Problem 7 (14 points)
Find the length of the curve given by $x = \frac{t^3}{3} - t$, $y = t^2$, for $0 \leq t \leq 2$.

Problem 8 (16 points)
Find the area inside one loop of the four-leafed rose $r = \cos(2\theta)$.

Problem 9 (20 points)

Let $\vec{u} = 2\vec{i} - 4\vec{j} + \sqrt{5}\vec{k}$ and $\vec{v} = -2\vec{i} + 4\vec{j} - \sqrt{5}\vec{k}$. Find:

(a) $\vec{u} \cdot \vec{v}$

(b) $|\vec{u}|$

(c) $|\vec{v}|$

(d) the cosine of the angle between $\vec{u}$ and $\vec{v}$

(e) the projection of $\vec{v}$ onto $\vec{u}$

Problem 10 (16 points)
Let $P = (2, 3, 4), Q = (3, 5, 7)$, and $R = (0, 3, 5)$.
(a) Find an equation for the plane containing P, Q, and R.

(b) What is the area of the *TRIANGLE* with vertices P, Q, and R?

Problem 11 (15 points)

A particle travels in space with velocity vector $\vec{v}(t) = 6t\vec{i} + 2\vec{j} + 14t\vec{k}$. At the time when $t = 1$ its position vector is $\vec{r} = 7\vec{i} + \vec{j} + 12\vec{k}$. What is its position vector $\vec{r}(t)$ in general?

Problem 12 (20 points)

Let $\vec{r}(t) = 6\sin(2t)\vec{i} + 6\cos(2t)\vec{j} + 5t\vec{k}$ define a path in space.

(a) Find the *unit tangent vector* $\vec{T}(t)$ to this path.

(b) Find the *unit normal vector* $\vec{N}(t)$ to this path.

(c) Find the *unit binormal vector* $\vec{B}(t)$ to this path.

(d) Find the *curvature* κ of this path.

(e) Find the *torsion* τ of this path.

SCRATCH PAPER