Your Name: ______________________________

Mathematics 221, Spring 2006 **Lecture 3 (Wilson)**

Final Exam May 7, 2006

Write your answers to the ten problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using $\frac{\pi}{3}$, $\sqrt{3}$, $\cos(0.6)$, $\ln(2)$), e^3, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

There is a problem on the back of this sheet: Be sure not to skip over it by accident!

There is scratch paper at the end of this exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought in, up to 3 sheets of notebook paper.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" (without more details) are not sufficient substantiation...)

Problem	Points	Score
1	24	
2	18	
3	24	
4	20	
5	16	
6	18	
7	18	
8	20	
9	22	
10	20	
TOTAL	100	

Problem 1 (24 points)

Find the derivative $D_x y$ for:

(a) $y = \sin(x)\,\cos(x)$

(b) $y = \dfrac{x^3}{\sin(x)}$

(c) $y = \sin^{-1}(x^2)$ (If you prefer the other notation, $y = \arcsin(x^2)$)

(d) $y = \ln[(2 - x)\,(1 + x^3)]$.

Problem 2 (18 points)
Find an equation for the tangent line to the graph of $y = \cos^{-1}(2x)$ (or in other notation $y = \arccos(2x)$, at the point $(\frac{1}{4}, \frac{\pi}{3})$.

Problem 3 (24 points)
Evaluate the integrals:

(a) $\int x^2 \cos(x^3 + 5)\, dx.$

(b) $\int \frac{dx}{4 + x^2}.$

Problem 4 (20 points)

The Fundamental Theorems of Calculus tell us relationships between the derivative and the integral. Using words, equations, pictures, whatever you find helpful, explain what these theorems say.
(I won't worry about which one you call the First theorem or the Second, different books don't even agree on that. There is no one right answer to this question! Try to describe what is going on, what the relationships between derivative and integral are that are so important.)

(a) One of the Fundamental Theorems of Calculus:

(b) The other Fundamental Theorem of Calculus:

Problem 5 (16 points)
Find the area between the curves $y = x^2$ and $y = 2x - x^2$.

Problem 6 (18 points)

Find all solutions of the equation $\frac{dy}{dx} + \frac{1}{x}y = 2e^{x^2}$.
You may assume $x > 0$.

Problem 7 (18 points)
Let $y = 2x^3 + 3x^2 - 36x - 4$.

(NOTE: There is another problem in which this same function appears. You may be able to save doing some work twice.)

(a) On which intervals of real numbers is this an increasing function? A decreasing function? Give reasons based on calculus, not just looking at the graph on a calculator.

(b) On which intervals of real number is the graph of this function concave upward? Concave downward? Give reasons based on calculus, not just looking at the graph on a calculator.

(c) Where does this graph have point(s) of inflection?

Problem 8 (20 points)
Let R be the region between the curves $y = x$ and $y = x^2$.
If we rotate R about the x-axis, what is the volume of the resulting solid?

Problem 9 (22 points)
Evaluate the integrals:

(a) $\displaystyle\int_1^2 \frac{e^{3/x}}{x^2}\,dx.$

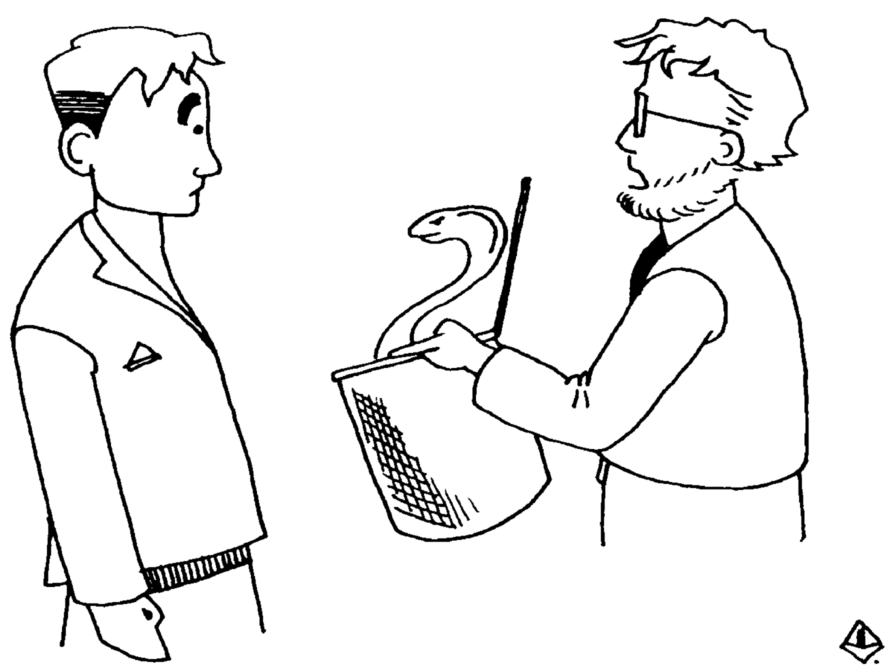
(b) $\displaystyle\int_0^{\frac{\pi}{4}} \sin^3(2x)\,\cos(2x)\,dx.$

Problem 10 (20 points)
Let $y = 2x^3 + 3x^2 - 36x - 4$.

(NOTE: There is another problem in which this same function appears. You may be able to save doing some work twice.)

Find and identify any/all local/global maxima and minima for this function.

SCRATCH PAPER



THE CHRONICLE OF HIGHER EDUCATION

JOHN LEAVITT

"You can take the final or battle the cobra."