Your Name: _____

Circle your TA's name:

George Brown | Carl Carlson | David Milovich

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Mathematics 221, Fall 2007 Lecture 1 (Wilson)

Second Midterm Exam November 29, 2007

Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using $\frac{\pi}{3}$, $\sqrt{3}$, $\cos(0.6)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

There is a problem on the back of this sheet: Be sure not to skip over it by accident!

There is scratch paper at the end of this exam. If you need more scratch paper, please ask for it.

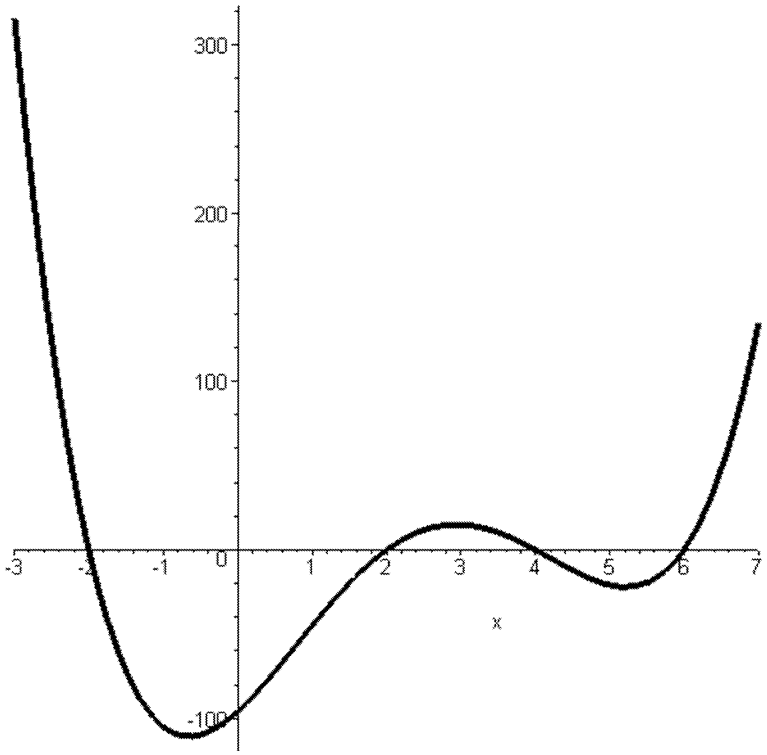
You may refer to notes you have brought in on an index card, as announced in class and at the class website.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" (without more details) are not sufficient substantiation...)

Problem	Points	Score
1	12	
2	12	
3	14	
4	12	
5	12	
6	15	
7	11	
8	12	
TOTAL	100	

Problem 1 (12 points)

At the right is the graph of a function $f(x)$, defined only for $-3 \leq x \leq 7$. Use that graph in answering the following.
Your answers may not be exact, reading from the graph, but come as close as you can.



(a) For what value(s) of x is $f(x) = 0$?

(b) For what value(s) of x is $f''(x) = 0$?

(c) On what interval(s) $[a, b]$ is $f(x)$ decreasing?

(d) On what interval(s) $[a, b]$ is the graph concave upward?

(e) At what value(s) x does $f(x)$ have a critical point?

(f) At what value(s) x does $f(x)$ have an inflection point?

Problem 2 (12 points)
Evaluate the integrals:
(Express your answers in terms of the original variable x.)

(a) $\int x \sqrt[4]{1-x^2}\, dx.$

(b) $\int (\sin(2x))^3 \cos(2x)\, dx.$

(c) $\int \cos^2 x\, dx$

Problem 3 (14 points)

Part I Let $f(x) = 2x^3 + 3x^2 - 12x + 4$ (for all numbers x).

(a) Does $f(x)$ have any absolute (global) maxima or minima? (Yes/No) Give a reason for your answer!

(b) Find all maxima and minima of f: Identify each as to maximum or minimum and local or absolute, and give reasons for your answers.

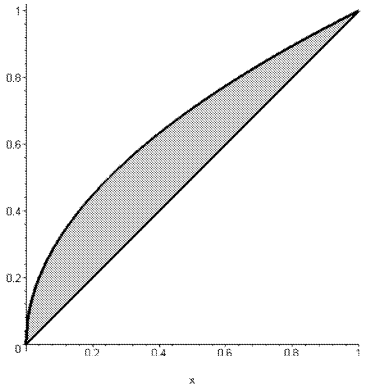
Part II Using the same formula $f(x) = 2x^3+3x^2-12x+4$ but this time restricting the domain so that f is only defined for $-1 \leq x \leq 2$:

(c) Does $f(x)$ have any absolute maxima or minima? (Yes/No) Give a reason for your answer!

(d) Find all maxima and minima of f: Identify each as to maximum or minimum and local or absolute, and give reasons for your answers.

Problem 4 (12 points)

The region between $y = x$ and $y = \sqrt{x}$ (for $0 \leq x \leq 1$) is shown at the right. This region is rotated about the y-axis to produce a solid which is conical on the outside and has a flared hole in the inside.



Set up and evaluate an integral to compute the volume of this solid.
You are free to choose whether to use "washers" or "shells".

Problem 5 (12 points)

(a) If $f(x) = 4x^2 - 3x + 1$, $a = 1$, and $b = 3$, find a number that works as c in the Mean Value Theorem for Derivatives, i.e. $f'(c) = \frac{f(b)-f(a)}{b-a}$.
Show how you found c, and work out both $f'(c)$ and $\frac{f(b)-f(a)}{b-a}$ to verify that your choice of c is correct.

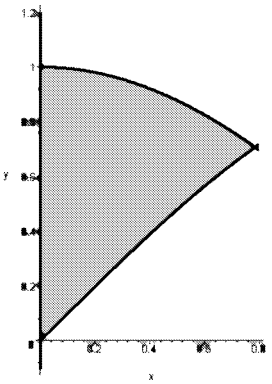
(b) Evaluate $\lim_{x\to 0} \dfrac{x - \sin x}{x^3}$.
Be sure to show why rules or theorems apply where you use them.

Problem 6 (15 points)

(a) Evaluate $\displaystyle\int_0^1 \frac{x+1}{(x^2+2x+2)^3}\,dx.$

(b) Evaluate $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan x \sec^2 x\,dx.$

(c) Set up and evaluate an integral to find the area of the "curved triangle" to the right of the y-axis, below $y = \cos x$, and above $y = \sin x$.



Problem 7 (11 points)

A rectangular box with a square base needs to be painted, but the bottom won't be painted. (So there are four sides plus the top that need to be painted.) The box needs to contain 500 cubic inches.

What should be the dimensions of the box in order to minimize the area that has to be painted? Be sure to give both the height of the box and the length of a side of the base.

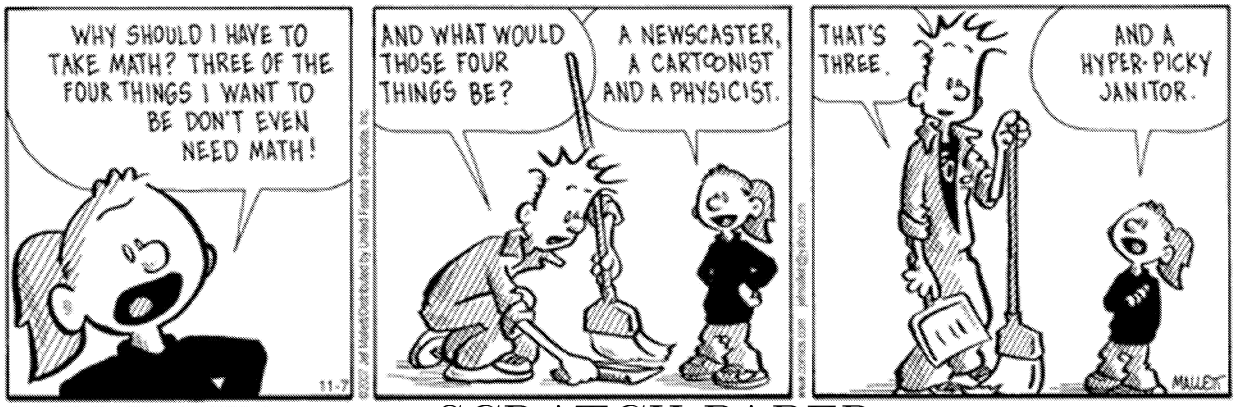
Show your work. Explain why you know your answer minimizes the area rather than maximizing it!

Problem 8 (12 points)
Solve the Initial Value Problem

$$f''(x) = 6x - 4 \text{ and } f'(1) = -1 \text{ and } f(0) = 2$$

(a) First find f': Use the fact that $f'' = 6x - 4$ to find an infinite collection of possible functions, then use $f'(1) = -1$ to select the right one.

(b) Now find f: Find a collection of functions that have for their derivatives whatever you got in (a), then use $f(0) = 2$ to select the right one.



SCRATCH PAPER

{not a command!}