Your Name: _____

Circle your TA's name:

Ben Akers   Adam Berliner   Ben Ellison

James Hunter   Peter Spaeth

Mathematics 221, Fall 2003   Lecture 2 (Wilson)

Second Midterm Exam   November 18, 2003

Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using $\frac{\pi}{3}$, $\sqrt{3}$, $\cos(0.6)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

There is a problem on the back of this sheet: Be sure not to skip over it by accident!

There is scratch paper at the end of this exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought in on one or two index cards, as announced at the class website.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" (without more details) are not sufficient substantiation...)

Problem	Points	Score
1	12	
2	13	
3	13	
4	12	
5	12	
6	12	
7	13	
8	13	
TOTAL	100	

<u>Problem 1</u>   (12 points)

Let $f(x) = x^3 - 6x^2 - 15x + 1$.

Find all local and global maxima and minima of $f(x)$ on the interval $[-3, 6]$.

For each answer, be sure to distinguish the x value and the value the function takes there, and tell how you know that this point is what you claim it is (local max, global max, local min, global min).

Problem 2   (13 points)
Let $f(x) = x + 2$.

(a) Set up and evaluate the Riemann sum for $f(x)$ which results from partitioning the interval $[-1, 1]$ into 4 equal-width subintervals and using the left end of each subinterval as the sample point.

(b) Set up and evaluate the Riemann sum for $f(x)$ which results from partitioning the interval $[-1, 1]$ into n equal-width subintervals (not for a specific value of n), and using the left end of each subinterval as the sample point. (Your answer when evaluating this sum should be a formula that involves n and possibly some summation signs Σ.)

(c) Evaluate the limit of the sum in (b) as n goes to ∞. This should give the same answer as $\int_{-1}^{1} (x + 2) \, dx$, and you can check your answer that way, but you must show how to get it from the limit of Riemann sums in order to receive credit for this problem.

<u>Problem 3</u>   (13 points)

Solve the initial value problem

$$\frac{dy}{dx} = \frac{x^2}{y}, \qquad \text{for } y > 0, \qquad\qquad \text{with} \qquad y(0) = 3.$$

Show explicitly the general solution to the differential equation and then how you pick the particular solution meeting the initial condition.
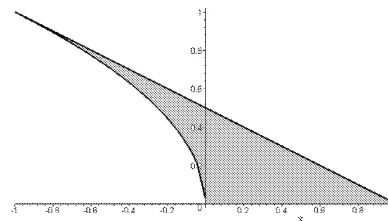
Use a linear approximation to $f(x) = \sqrt[3]{x}$ to approximate $\sqrt[3]{7}$.

Hint: The tangent line to the graph of $f(x)$ at $x = 8$ is particularly easy to work with.

Problem 5   (12 points)

(a)   Evaluate $\displaystyle\int_{-\frac{\pi}{4}}^{0} \sin^2(2x)\,\cos(2x)\,dx$.

(b) Find the area of the region shown, bounded by parts of the curve $x = -y^2$, the line $2y = 1-x$, and the x-axis.

Let $f(x) = x^2 - 6x + 7$.

(This same function is used in both (a) and (b) below!)

  (a) Find the average (mean) value of $f(x)$ on the interval $[2, 5]$.

  (b) The Mean Value Theorem for Integrals guarantees the existence of a number c <u>within</u> the interval $[2, 5]$ (i.e. c is not either of the endpoints) such that $f(c)$ is the average value. Find such a number c.

Problem 7   (13 points)

The region between the graphs of $y = \sqrt{x}$ and $y = x^2$ is shown at the right. Set up and evaluate an integral to compute the volume of the solid that results when this region is rotated about the x-axis.
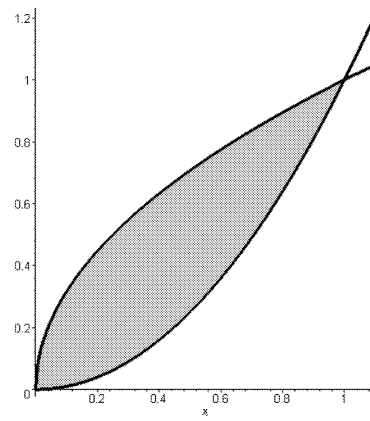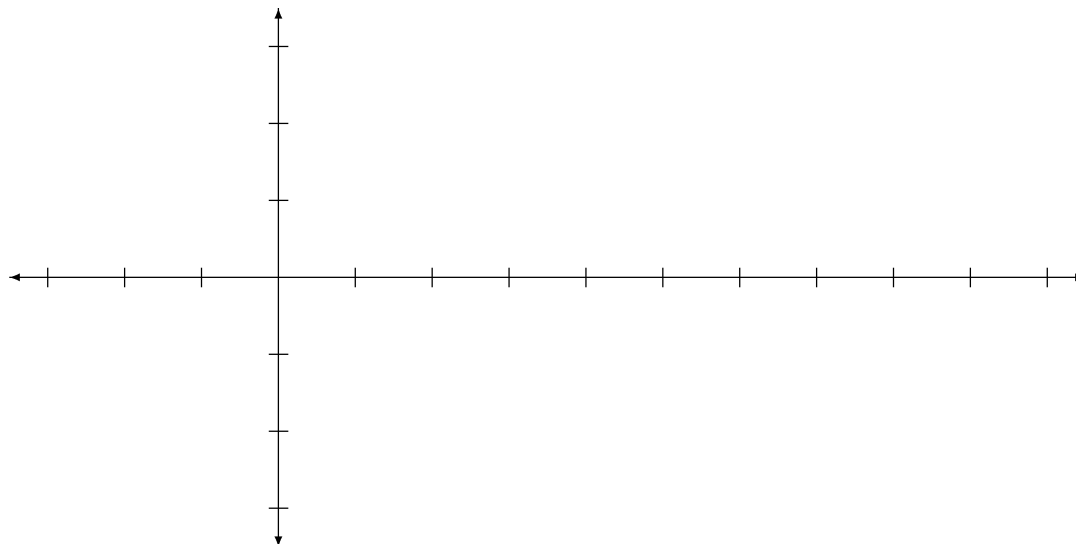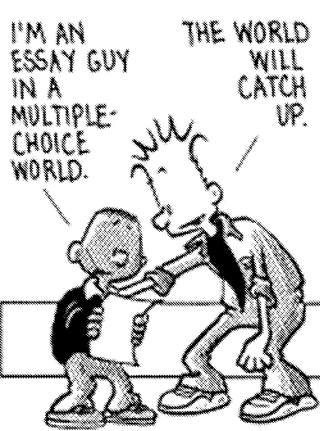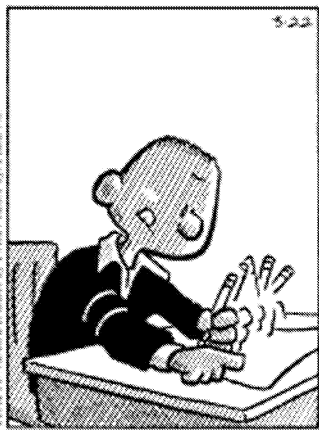
Problem 8   (13 points)

Suppose $f(x)$ is a function with the following properties:

- $f(0) = 2$

- $f'(x) < 0$ for $-3 \leq x < 0$
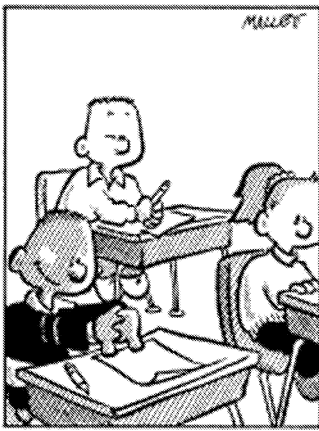
- $f'(0) = 0$

- $f'(x) < 0$ for $0 \leq x < 6$

- $f'(x) > 0$ for $6 < x \leq 10$

- $f''(x) > 0$ for $-3 \leq x < 0$

- $f''(0) = 0$

- $f''(x) < 0$ for $0 < x < 4$

- $f''(4) = 0$

- $f''(x) > 0$ for $4 < x \leq 10$

Draw on the axes below the graph of a function with these properties.

SCRATCH PAPER