Math 635 – Spring 2010

Homework 1

Due: February 2, 2010, beginning of the class. Late homework will **not** be accepted.

1. Consider a probability space with infinitely many coin flips $X_1, X_2, X_3, \ldots$. (I.e. each X_i is 0 or 1.) Consider the sequence of σ-fields $\{\mathcal{F}_n\}$ generated by these random variables. Describe the elements of $\mathcal{F}_3$.

2. Suppose that X_i are i.i.d random variables with $EX_i = 0, EX_i^2 = 1, EX_i^3 = 0$ and let $S_n = \sum_{i=1}^n X_i$. Find a cubic polynomial $f(x)$ (with n-dependent coefficients) such that $\{f(S_n)\}$ is a martingale with respect to $\{X_n\}$.

3. Suppose that τ_1 and τ_2 are stopping times with respect to $\{\mathcal{F}_n\}$. Show that $\tau_1 + \tau_2$ is also a stopping time.

4. Consider the gambler's ruin problem with $A = B$, let τ be the first time we reach $\pm A$. Compute the moment generating function of τ, i.e. $Ee^{\lambda\tau}$.
   Hints: modify the sequence of random variables e^{tS_n} so that you get a martingale and consider the stopped martingale. You will need to use the fact that by symmetry S_τ is a random variable which is $\pm A$ with probability 1/2-1/2, and it is independent of τ.