

Homework 4

Due: October 4, 2012, beginning of the lecture

PLEASE READ THE INSTRUCTIONS/SUGGESTIONS WRITTEN IN THE SYLLABUS!

Problems from the textbook:

- 1.11, Page 63: 1 a), c), 2 a), b), c), 4 a), b), c), d), 8
- 1.15, Page 70: 1, a), b), c), d) 2, 3, 4a, 5, 7a.

BONUS PROBLEM: The sequence of non-negative integers $a_1 = 1, a_2, a_3, \dots$ has the following property:

$$a_{n+1} \leq 1 + a_1 + a_2 + \dots + a_n \quad \text{for } n \geq 1$$

Prove that *any* positive integer can be expressed as a sum of a couple of elements from this sequence. (Notes: we might have a sum of a single term. If the sum has more than one terms then they have to be elements from the sequence with different indices.)

Example: the sequence $a_n = 1$ for all n satisfies the conditions, and any natural number k may be expressed as a valid sum: $k = a_1 + a_2 + \dots + a_k$. Of course, you have to prove the statement for all possible sequences satisfying the condition.)

DISCLAIMER: It is easy to find the solutions to (some of) these questions. (E.g. the internet, your fellow classmates ...) However, do NOT consult any of these solutions when working on this assignment or you will learn nothing from it and your chance of passing the course will be greatly diminished. If it becomes apparent to the grader that your solution is copied from existing solutions, you will be assigned a grade of zero for lack of originality.