Sample Final Exam

1. Consider the function $z = \ln(x^2 + y^2 - 4)$.
   - Find the domain and the range.
   - Describe the level sets of the function.
   - Find the equation for the tangent plane at the point $(2, 2, \ln 4)$
2. Find the absolute extrema of $f(x, y) = x^2 + 3y^2 + 2y$ on the closed ellipse $x^2 + 2y^2 \leq 1$.
3. Convert to cylindrical coordinates and then evaluate the following integral:

$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-(x^2+y^2)}^{x^2+y^2} xy^2 dzdydx$$

4. Evaluate the line integral $\int_C y^2 dx + x^2 dy$ where C is the quarter of the circle $x^2 + y^2 = 4$ lying in the first quadrant with starting point: $(2, 0)$ and end point $(0, 2)$.
5. Parametrize the spherical band given by $x^2 + y^2 + z^2 = 36$ with $-3 \leq z \leq 3\sqrt{3}$ and use this to compute the surface area.
6. One of these vector fields are conservative. Decide which one it is and compute the flow integral on a path connecting $(0, 0, 1)$ with $(1, 2, 3)$.
   (a) $\mathbf{F}(x, y, z) = (xe^y, ye^z, ze^x)$
   (b) $\mathbf{G}(x, y, z) = (ye^{xy+z^2}, xe^{xy+z^2}, 2ze^{xy+z^2})$
7. Use Stokes' theorem to compute the circulation of the curve C in the vector field $\mathbf{F}(x, y, z)$ where C is the intersection of the cylinder $x^2 + y^2 = 4$ and the hemisphere $x^2 + y^2 + z^2 = 16, z \geq 0$ and $\mathbf{F}(x, y, z) = (x^2y^3, 1, z)$.
8. State the Divergence Theorem.