INTEGRAL FUNCTIONALS, OCCUPATION TIMES AND HITTING TIMES OF DIFFUSIONS

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In this talk several known and some new identities between integral functionals, occupation times and hitting times of diffusions and, especially, Brownian motion are discussed. As an example (due to Dufresne and Yor) consider

$$\int_0^\infty \exp(-2B_s^{(\mu)})\, ds \overset{(d)}{=} H_0(R^{(\delta)})$$

where $B^{(\mu)}$ is a Brownian motion with drift $\mu > 0$ started from 0, $R^{(\delta)}$ is a Bessel process of dimension $\delta = 2(1-\mu)$ started from 1, and

$$H_0(R^{(\delta)}) = \inf\{s > 0 : R_s^{(\delta)} = 0\}.$$

Another example (due to Biane and Imhof) is

$$\int_0^\infty \mathbf{1}_{\{B_s^{(\mu)}<0\}})\, ds \overset{(d)}{=} H_\lambda(B^{(\mu)})$$

where $B^{(\mu)}$ is as above and λ is an exponentially with parameter 2μ distributed random variable independent of $B^{(\mu)}$.

It is well known that the Feynman-Kac formula is a very powerful tool to compute the distributions of functionals of linear diffusions but this formula does not usually explain identities between different functionals. However, in many cases such identities can be better understood using random time change and Ray-Knight theorems, as is demonstrated in this talk which is based on the following papers:

1. Borodin, A.N. and Salminen, P. (2004). On some exponential integral functionals of BM(μ) and BES(3). *Zap. Nauchn. Semin. POMI* **311**, 51–78.
2. Kozlova, M. and Salminen, P. (2005). On an occupation time identity for reflecting Brownian motion with drift. *Periodica Math. Hung.* (to appear).
3. Kozlova, M. and Salminen, P. (2005). A note on occupation times of stationary processes. *ECP* (to appear).
4. Salminen, P. and Yor, M (2005). Properties of perpetual integral functionals of Brownian motion with drift. *Ann. I.H.P.,* (to appear).
5. Salminen, P. and Yor, M (2005). Perpetual integral functionals as hitting times and occupation times. *EJP* (to appear).