632 Introduction to Stochastic Processes Fall 2008
Part of Homework 2

1. You are trying to cross a busy highway. You need time c to cross safely. Interarrival times between cars are i.i.d. with common distribution $p_k = pq^{k-1}$ for $k \in \mathbf{N}$. Find the expected time of your arrival on the other side of the road.

A little more precisely: You arrive at the side of the road at time 0. Let $S_0 = 0$. The arrival times of cars are S_1, S_2, $S_3, \ldots$ where $S_n = X_1 + \cdots + X_n$ and the random variables $\{X_i\}$ are i.i.d. with $P(X_i = k) = pq^{k-1}$ for positive integers k. Assume $0 < p < 1$, $q = 1 - p$, and let c be a positive integer. Let

$$N = \inf\{n \geq 1 : X_n \geq c\}$$

mark the first interarrival time of length at least c. As soon as the previous car has passed you cross the road and arrive to the other side at time $T = S_{N-1} + c$. Find $E(T)$ in terms of the two parameters of the model, p and c.

If you use Wald's identity, be sure to check its hypotheses.

After you found an answer, try to check that it makes sense. For example if $c = 1$ the answer should be pretty obvious. Does your formula give the right answer?