Math 629 Introduction to Measure and Integration Spring 2006
Homework 7

Due Monday, April 24

1. (a) Let $0 \leq X \leq \infty$ be a random variable. Show that

$$E(X) = \int_0^\infty P(X > s)\, ds = \int_0^\infty P(X \geq s)\, ds.$$

Hint: Write $X = \int_0^X ds$ and use Tonelli's theorem.

(b) Let $\{X_n : n \in \mathbb{N}\}$ be identically distributed random variables on $(\Omega, \mathcal{F}, P)$. The hypothesis means that $P(X_k \in B) = P(X_n \in B)$ for all $k, n \in \mathbb{N}$ and $B \in \mathcal{B}_\mathbb{R}$. Assume $E\,|X_1| < \infty$. Show that then $X_n/n \to 0$ almost surely as $n \to \infty$.

Hints: This is not an exercise in the law of large numbers because no independence is assumed. This will follow from the right use of part (a) and one of the techniques we used to prove the Strong Law of Large Numbers.

2. Exercise 4 on page 88.

3. Exercise 7 on page 88.