Math 629 Introduction to Measure and Integration Spring 2006
Homework 5

Due Monday, April 3

When nothing else is specified, the exercise takes place on some abstract measure space $(X, \mathcal{M}, \mu)$.

1. Given a function $f : \mathbb{R} \to \mathbb{R}$ and $h \in \mathbb{R}$, define the translated function f_h by $f_h(x) = f(x + h)$. Suppose f is Borel measurable and integrable under Lebesgue measure. Show that then f_h is also Borel measurable and integrable. Show that this translation operation is continuous in the following sense:

$$\lim_{h \to 0} \int_{\mathbb{R}} |f_h - f| \, dx = 0.$$

2. Exercise 34 on page 63. Notice that what you are proving here is a dominated convergence theorem under an assumption of convergence in measure. (It is not necessary to prove part a. first.)