Math 629 Introduction to Measure and Integration Spring 2006
Homework 2

Due Friday, Feb. 17

1. Exercise 18 on p. 32.

2. (a) Use the previous exercise to show that in the setting of Theorem 1.14, if μ_0 is σ-finite, the measure space $(X, \mathcal{M}^*, \mu^*)$ is the completion of the measure space $(X, \sigma(\mathcal{A}), \mu)$. As in class, $\mathcal{M}^*$ denotes the collection of μ^*-measurable sets defined as on p. 29, in terms of the outer measure (1.12).

(b) Give an example where a premeasure μ_0 on an algebra $\mathcal{A}$ has more than one extension to a measure on $\sigma(\mathcal{A})$. Make it as simple as possible. You should be able to do it on the space $X = \mathbb{Z}_+$ of nonnegative integers, say.

3. Prove this highly useful approximation property: Let $(X, \mathcal{M}, \mu)$ be a finite measure space (means that $\mu(X) < \infty$), and assume the σ-algebra $\mathcal{M}$ is generated by the algebra $\mathcal{A}$. Then given $E \in \mathcal{M}$ and $\varepsilon > 0$, there exists a set $A \in \mathcal{A}$ such that $\mu(A \triangle E) < \varepsilon$.