521 Analysis I Spring 2011
Quiz 2

1. (a) Let $\{x_n\}$ be a real sequence. State the definition of the property that x_n converges to ∞.

In the next two parts, be sure to justify fully what you claim.

(b) Let $s_n = n^2 + 2011 \cdot (-1)^n$. Using the definition from part (a), prove or disprove that $s_n \to \infty$.

(c) Let $t_n = 1 + \frac{6}{n+5}$. Using the definition from part (a), prove or disprove that $t_n \to \infty$.

(d) I have a real sequence $\{y_n\}$ that is Cauchy and converges to infinity. What can you say about such a sequence? Justify your claim by appeal to results we know.

(a) $x_n \to \infty$ if for every $M < \infty$ there exists $N < \infty$ s.t. $n \geq N \implies x_n \geq M$.

(Note: don't use ε in place of M here. ε is used for small quantities.)

(b) $s_n \geq n^2 - 2011$. $n^2 - 2011 \geq M \iff n \geq \sqrt{M + 2011}$.
So $N = \sqrt{M + 2011}$ (or the next integer after that) works to give $n \geq N \implies s_n \geq M$. So $s_n \to \infty$.

(c) $n \geq 1 \implies t_n \leq 1 + \frac{6}{1+5} = 2$. So for example if $M = 3$ then $t_n < M$ for all n, and we cannot satisfy the def.
So $t_n \not\to \infty$.

(d) Every Cauchy sequence in $\mathbb{R}$ converges to a limit in $\mathbb{R}$, hence this kind of a sequence does not exist.