SKETCH

521 Analysis I Spring 2011
Quiz 1

Be sure to justify your answers. Recall this definition: the closure $\bar{A}$ of a set A in a metric space is defined as $\bar{A} = A \cup A'$ where A' is the set of limit points of A.

1. In a metric space (X, d) define the distance of a point x to the set A as

$$\mathrm{dist}(x, A) = \inf\{d(x, a) : a \in A\}.$$

(In other words, the distance from x to A is the infimum of all the distances from x to points of A.)

(a) Show that if $x \in A$ then $d(x, a) = 0$.

(b) Take the space $X = \mathbb{R}$ with its usual metric $d(x, y) = |x - y|$ and let $B = \{\frac{1}{n} : n \in \mathbb{N}\} = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\}$. Find $\mathrm{dist}(0, B)$.

(c) In a general metric space, show that if $x \in \bar{A}$ then $\mathrm{dist}(x, A) = 0$.

Since $d(x,a) \geqslant 0$ always, also $\mathrm{dist}(x,A) \geqslant 0$.

(a) $\mathrm{dist}(x,A) \leq d(x,x) = 0$. Together with above, $\mathrm{dist}(x,A) = 0$.

(c) 0 is always a l.b. for the set $\{d(x,a) : a \in A\}$.

To show no $r > 0$ can be a l.b.:

$x \in \bar{A} \implies \exists\, y \in A \cap N_r(x)$.

$d(x,y) < r \implies r$ cannot be a l.b.

Consequently $\mathrm{dist}(x,A) = \inf\{d(x,a) : a \in A\} = 0$.

[(b) is similar to (c); in fact a special case.]