Ch 6 Sketches

2. Suppose $\exists x_0 \in [a,b]$ s.t. $f(x_0) > 0$. Use continuity to show that $\exists \alpha > 0, \delta > 0$ s.t. $f(x) \geq \alpha$ for $x \in [x_0 - \delta, x_0 + \delta] \cap [a,b]$. Take any partition P with mesh $< \delta$. Then $\exists$ interval $[x_{i-1}, x_i] \subseteq [x_0 - \delta, x_0 + \delta]$, and then $L(P) \geq \alpha \cdot \Delta x_i$. Since the integral is assumed to exist, $\int_a^b f\,dx \geq L(P) > 0$, contradiction.

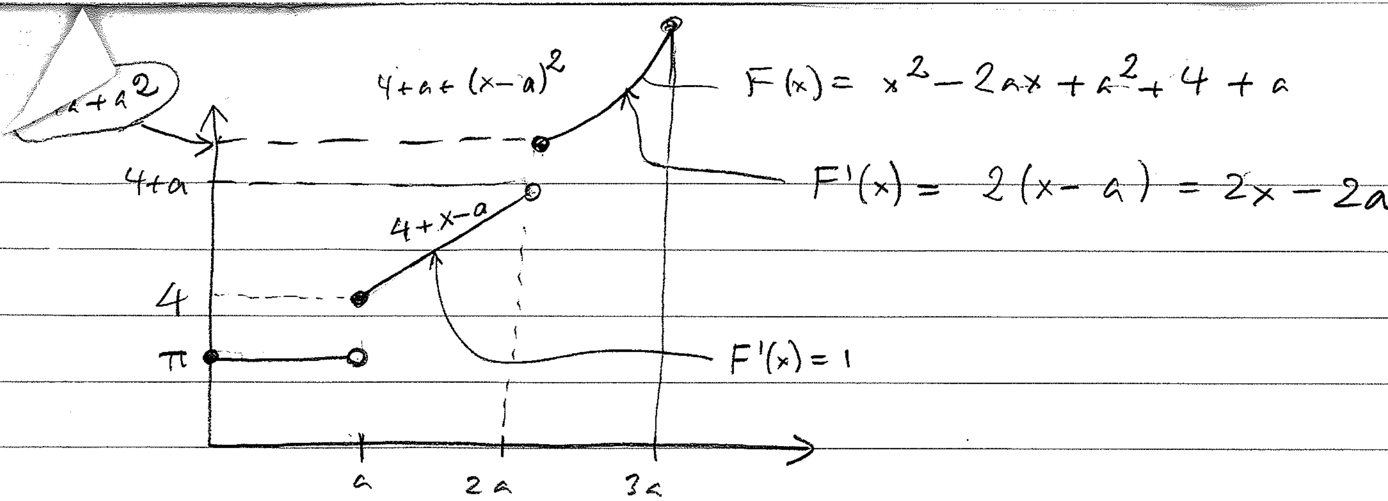
8. The ingredients: $a_n = \sum_{k=1}^{n} f(k)$ is a nondecreasing sequence, $F(t) = \int_1^t f(x)\,dx$ is a nondecreasing function. Thus limits as $n \to \infty$ or $t \to \infty$ exist if a_n or $F(t)$ remains ~~bounded~~.
Justify the inequalities

$$\sum_{k=2}^{n} f(k) \leq \int_1^n f(x)\,dx \leq \sum_{k=1}^{n} f(k).$$

(For example, a picture can show this.)

4. Should be clear from looking at $L(P)$ and $U(P)$.
(Note: the Lebesgue integral does integrate this function!)

HW9 extra



$$\int_0^{3a} x\,dF(x) = (4-\pi)a + \int_a^{2a} x\,dx + \int_{2a}^{3a} x(2x-2a)\,dx + 2a\cdot a^2$$

$$= (4-\pi)a + \Big|_a^{2a} \frac{x^2}{2} + \underbrace{\int_{2a}^{3a} (2x^2-2ax)\,dx}_{\Big|_{2a}^{3a}\left(\frac{2}{3}x^3 - ax^2\right)}$$

$$= (4-\pi)a + a^2\left(2-\frac{1}{2}\right) + \underbrace{a^3\left(\cancel{18} - \frac{16}{3} - 9 + 4\right) + 2a^3}_{a^3\left(15-\frac{16}{3}\right) = a^3\left(\frac{29}{3}\right)}$$

$$= (4-\pi)a + \frac{3}{2}a^2 + \frac{29}{3}a^3$$