8. f' cont. on $[a,b] \Rightarrow f'$ unif cont on $[a,b]$. Given $\varepsilon > 0$, find $\delta > 0$ s.t. $x, y \in [a,b]$ and $|x-y| < \delta$ imply $|f'(x) - f'(y)| < \varepsilon$.
Now if $t, x \in [a,b]$, MVT implies the existence of p between t and x s.t. $f(t) - f(x) = f'(p)(t-x)$.
If $|t-x| < \delta$ then also $|x-p| < \delta$, and so

$$\left| \frac{f(t)-f(x)}{t-x} - f'(x) \right| = |f'(p) - f'(x)| < \varepsilon.$$

11. $$\lim_{h \to 0} \frac{f(x+h)+f(x-h)-2f(x)}{h^2} \underset{\text{l'Hospital}}{=} \lim_{h \to 0} \frac{f'(x+h)-f'(x-h)}{2h}$$

$$= \frac{1}{2} \cdot \lim_{h \to 0} \left[\frac{f'(x+h)-f(x)}{h} + \frac{f'(x-h)-f(x)}{-h} \right] = \frac{f''(x)+f''(x)}{2} = f''(x).$$

Example $$f(x) = \begin{cases} \frac{x^2}{2}, & x \geq 0 \\ -\frac{x^2}{2}, & x \leq 0 \end{cases} \qquad f'(x) = \begin{cases} x, & x \geq 0 \\ -x, & x \leq 0 \end{cases}$$

(Existence of $f'(0)$ needs separate check.) $f''(0)$ does not exist.