1 (a) Pf by contradiction: Suppose $x > a$.
Then we can find a number c s.t. $a < c < x$.
We have contradicted the assumption
$c > a \Rightarrow x \leq c$.

(b) Let $c > a$. Pick $\varepsilon > 0$ s.t. $c - \varepsilon > a$.
Pick N s.t. $n \geq N \Rightarrow |s_n - s| \leq \varepsilon$.
Then $s \leq s_n + \varepsilon \leq a + \varepsilon \leq c$.
Since $s \leq c$ for each $c > a$, we
have $s \leq a$.

$s = \overline{\lim}\, s_n$. $\quad t = \inf\limits_{\ell \in \mathbb{N}} \left(\sup\limits_{n \geq \ell} s_n \right)$. Claim: $s = t$

Claim 1. $t \leq s$.

If $s = \infty$ then nothing to prove.
Suppose $s < \infty$. Let $c > s$. Then Thm 3.17(b) implies that $\exists N \in \mathbb{N}$ s.t. $n \geq N \Rightarrow s_n < c$.
Hence
$$t = \inf_{\ell \in \mathbb{N}} \left(\sup_{n \geq \ell} s_n \right) \leq \sup_{n \geq N} s_n \leq c.$$

We have shown that $c > s \Rightarrow t \leq c$.
It follows that $t \leq s$ because if $t > s$ then we can find c s.t. $s < c < t$.

Claim 2. $t \geq s$.

Let $y_\ell = \sup\limits_{n \geq \ell} s_n$. We show that each y_ℓ is an upper bound of the set $E = \{x \in [-\infty, \infty] : \exists\, s_{n_k} \to x\}$.
Let $x \in E$. Then $s_{n_k} \to x$ for some subsequence $\{s_{n_k}\}$. Since $s_{n_k} \leq y_\ell$ for all large enough k (namely, once k is large enough to make $n_k \geq \ell$), in the limit also $x \leq y_\ell$.
(If $x > y_\ell$, then $s_{n_k} \to x$ implies $s_{n_k} > y_\ell$ for all large enough k, contradicting $y_\ell = \sup\limits_{n \geq \ell} s_n$.)

Thus now we have that $y_\ell = \sup_{n \geq \ell} s_n$ is an upper bound of E. By def., $s = \sup E$, so $s \leq y_\ell$.

The inequality $s \leq y_\ell$ is now true for all $\ell \in \mathbb{N}$. This says that s is a lower bound of the set $\{y_\ell : \ell \in \mathbb{N}\}$. But by def. $t = \inf_{\ell \in \mathbb{N}} y_\ell$, and hence $s \leq t$.