Case: $[p,q] \not\subseteq G$. Since $p \in G$, $p < q$, and $[p,q] \not\subseteq G$ then p is a lower bound of V_q.

Since $p \in G$, and G is open, there exists a neighborhood $N_r(p)$ around p such that $N_r(p) \subseteq G$. Then, pick a point $x \in N_r(p), x > p$. Then $[p,x] \subseteq G$, so $[x,q] \not\subseteq G$. Thus x is also a lower bound of V_q. This process can be repeated for every $x \in V_p$, meaning every $x \in V_p$ is not in the open inverval V_q. Therefore, the intervals V_p and V_q are disjoint.

Union of all such intervals covers G. Take $x \in G$. Since G is open, there exists a neighborhood $N_r(x) \subseteq G$. Pick $y \in N_r(x), y > x$. By Theorem 1.20, we can find $q \in \mathbb{Q}$ such that $x < q < y$. Since q also in $N_r(x)$, then $[x,q] \subseteq G$. Thus x is in a largest open interval around a rational. Since the choice of x was arbitrary, all $x \in G$ are covered by such intervals.

The number of such intervals is at most countable. Since the set $\mathbb{Q}$ is countable, every $x \in G$ is covered by a largest open interval around a rational, and all such intervals are disjoint, then the set $\mathbb{R}$ can be covered by at most countable, disjoint open intervals. $\square$