521 Analysis I Spring 2011 Homework 10 extra problem

Problem 10.1 (Schwarz inequality for integrals.) Let α be a nondecreasing function on [a, b] and f, g bounded functions on [a, b] that are members of $\mathcal{R}(\alpha)$. Show that then

$$\int_{a}^{b} |fg| \, d\alpha \le \left(\int_{a}^{b} f^2 \, d\alpha\right)^{1/2} \left(\int_{a}^{b} g^2 \, d\alpha\right)^{1/2}.$$