Biomixing and large deviations

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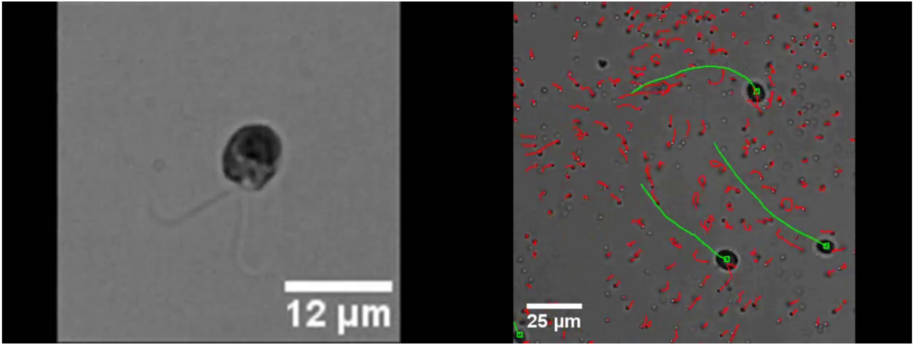
Chlamydomonas reinhardtii

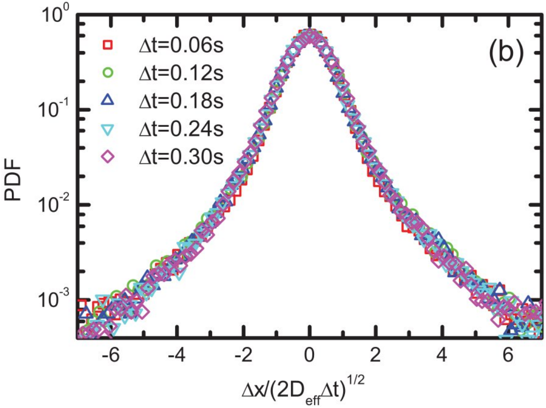


play movie

[Guasto, J. S., Johnson, K. A., & Gollub, J. P. (2010). *Phys. Rev. Lett.* **105**, 168102]

Probability density of displacements

Non-Gaussian PDF with ‘exponential’ tails:



[Leptos, K. C., Guasto, J. S., Gollub, J. P., Pesci, A. I., & Goldstein, R. E. (2009). *Phys. Rev. Lett.* **103**, 198103]

Probability density of displacements

Leptos *et al.* (2009) claim a reasonable fit of their PDF with the form

$$P_{\Delta t}(\Delta x) = \frac{1-f}{\sqrt{2\pi}\,\delta_g}\,\mathrm{e}^{-(\Delta x)^2/2\delta_g^2} + \frac{f}{2\delta_e}\,\mathrm{e}^{-|\Delta x|/\delta_e}$$

They observe the scalings $\delta_g \sim A_g(\Delta t)^{1/2}$ and $\delta_e \sim A_e(\Delta t)^{1/2}$, where A_g and A_e depend on ϕ.

They call this a diffusive scaling, since $\Delta x \sim \Delta t^{1/2}$. Their point is that this is strange, since the distribution is not Gaussian.

Commonly observed in diffusive processes that are a combination of trapped and hopping dynamics (Wang *et al.*, 2012).

Displacement by a moving body

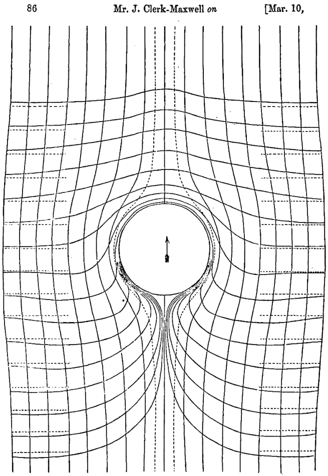


Fig. 1.
Fluid flowing past a fixed cylinder.

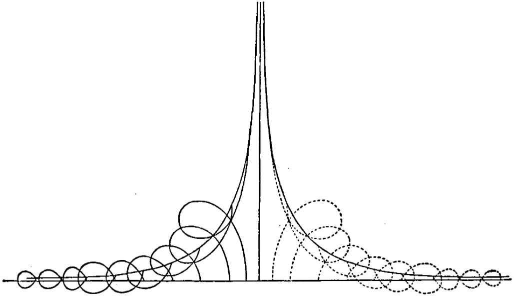


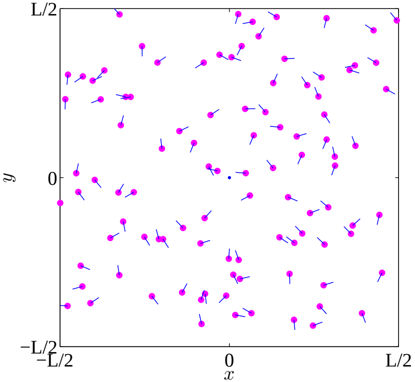
Fig. 2.
Paths of particles of the fluid when a cylinder moves through it.

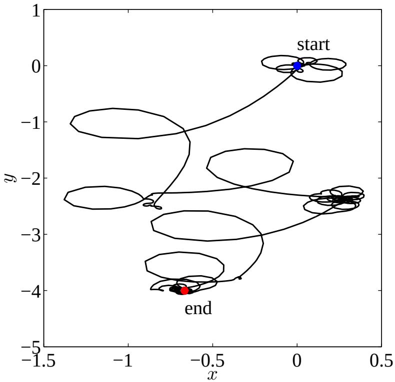
Maxwell (1869); Darwin (1953); Eames *et al.* (1994)

Numerical simulation

- Validate theory using simple simple simulations;
- Large periodic box;
- N_{swim} swimmers (cylinders of radius 1), initially at random positions, swimming in random direction with constant speed $U = 1$;
- Target particle initially at origin advected by the swimmers;
- Since dilute, superimpose velocities;
- Integrate for some time, compute $|\mathbf{x}(t)|^2$, repeat for a large number N_{real} of realizations, and average.

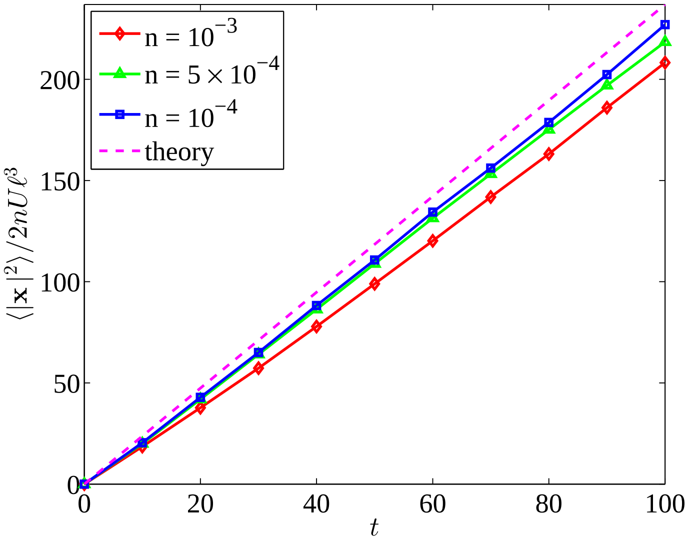
A 'gas' of swimmers





play movie 100 cylinders, box size = 1000

The dilute theory



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