J-L Thiffeault, Madison math circle

9/12/16

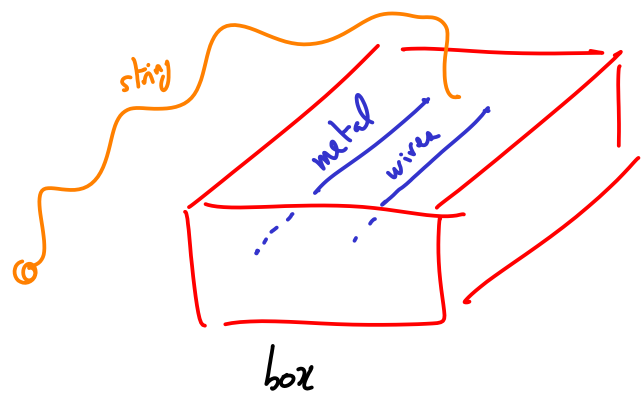
Why do my earbuds keep getting entangled?

Examples: earbuds, power cords, string

This is pretty complicated: to understand things mathematically, we need a model: a way to link the real and mathematical worlds

Usually, models involve some amount of simplification and idealization.

Model:

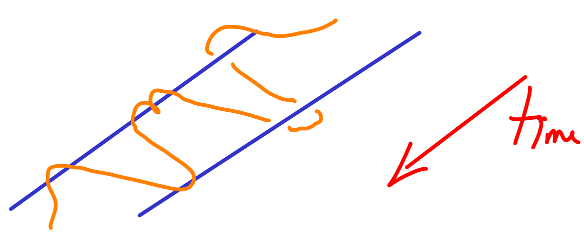


Ideally, get rid of sides of the box!

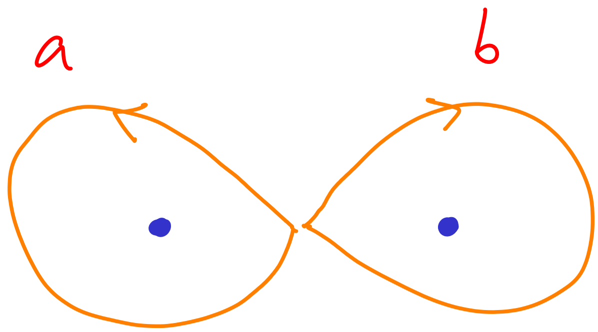
→ Show gets more entangled when we randomly flip.
→ show difficult to untangle. (Tangle manually if needed)

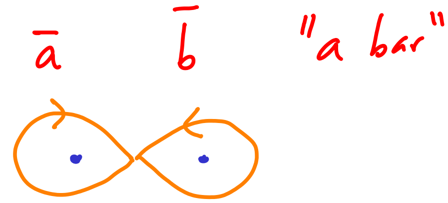
So this simple apparatus captures something of the real system

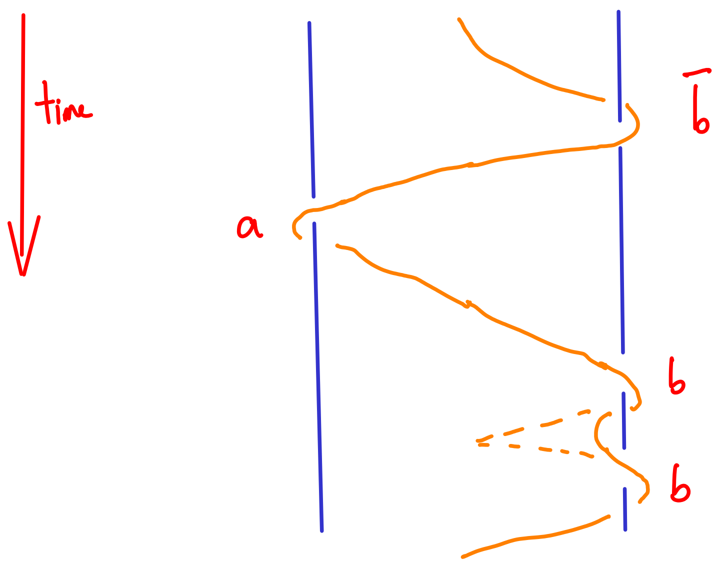
Eventually,



Now we ready to translate this to math







This entanglement is written

$\bar b a b b$

When we see $\bar a a$ or $\bar b b$ or $a \bar a$ or $b \bar b$, we CANCEL the two symbols:

$b \bar a a b = b b$

cancel!

Think of the cancellation as <u>detangling</u>!

The longer the sequence, the more entangled the string.

$a b \bar a b a b b a$ is more entangled than $\bar b a b$

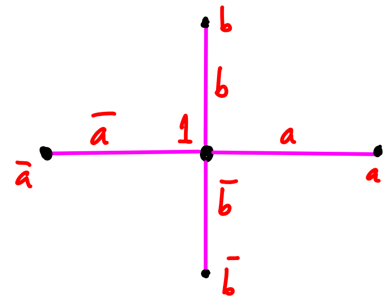
(8) (3)

The "unentangled state" we write as 1.

Now lets start with the unentangled state 1.

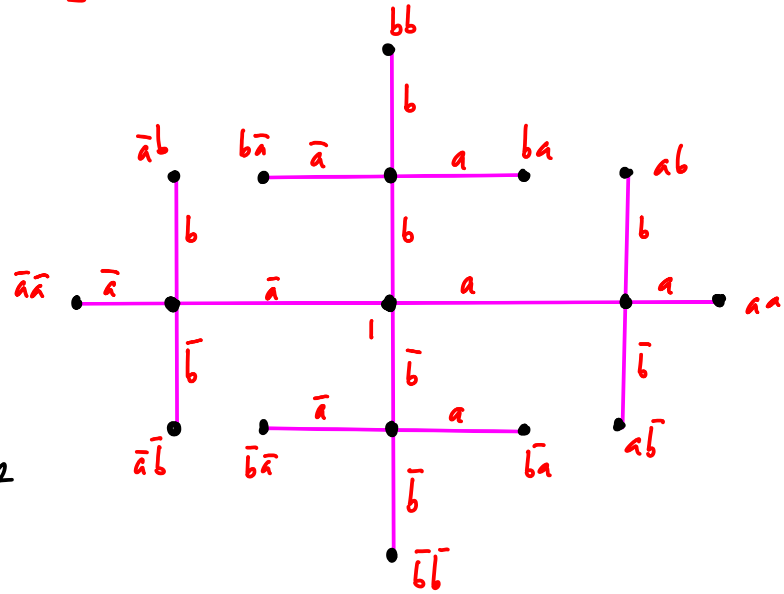
What can happen next? Well, a, b, $\bar{a}$, $\bar{b}$ can happen.

Let's represent this as a graph:



What next? aa
ab
Again, can get $a\bar{a} = 1$
4 symbols: $a\bar{b}$

"Cayley graph"



Goes on to infinity!

Every dot •
(vertex) has
4 exits.
One exit corresponds
to detangling

Now we have a nice model of entanglement: start at 1, roll a 4-sided die labeled $a, b, \bar{a}, \bar{b}$. Then move in the graph according to the die. Each has "probability" $1/4$.

Question: after rolling the die many times, how likely am I to return to 1, the unentangled state?

Answer: if we roll the die N times, the probability is

$$\left(\frac{1}{3}\right)^N$$

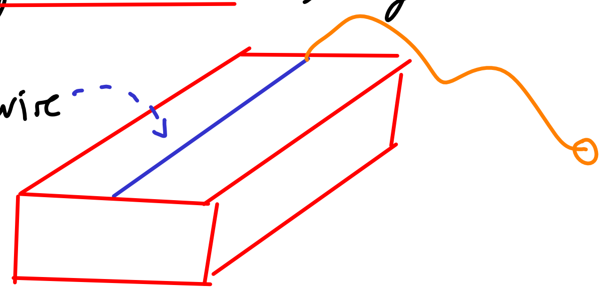
example: $N = 10$, $\left(\frac{1}{3}\right)^{10} = \frac{1}{3^{10}} = \frac{1}{59049}$!

And it only gets smaller as we increase n.

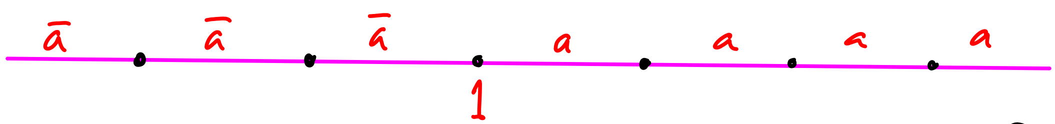
Hence, almost never "spontaneously" untangles

Question asked after lecture: why two wires? Great question!

With only one wire



We only need two symbols: a, $\bar{a}$



Then the "Cayley graph" as above is just a line

Can show: we will always eventually return to 1 (unentangled)!

Conclude: one wire not enough to model "complex" entanglement.

Important application: DNA, topoisomerase

(If time: knots, braids)