# Making taffy with the Golden mean

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# Making candy by hand


[movie 1] http://www.youtube.com/watch?v=pCLYieehzGs

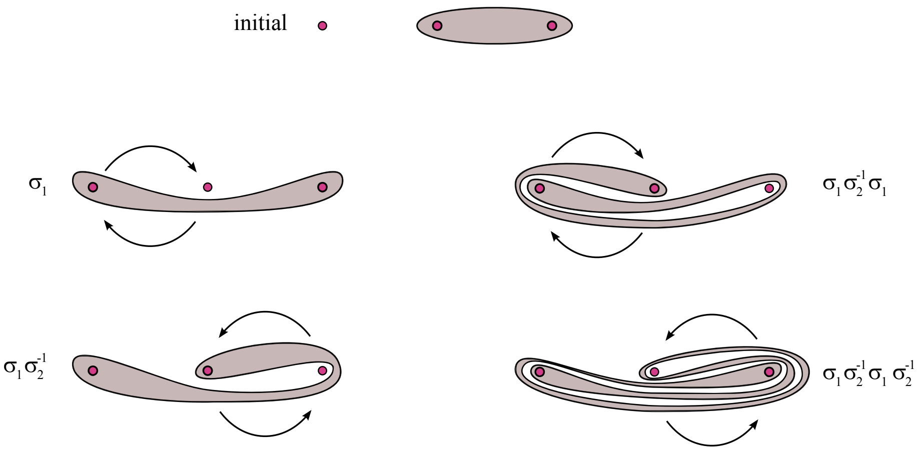
# Taffy pullers

[movie 2] [movie 3] http://www.youtube.com/watch?v=YPP2_Zf0IVU

# Four-pronged taffy puller

[movie 4] `http://www.youtube.com/watch?v=Y7tlHDsquVM`

# A simple taffy puller



# Number of folds

[Matlab: `demo1`]

The number of left/right folds satisfies:

$$\#\text{folds}_n = \#\text{folds}_{n-1} + \#\text{folds}_{n-2}$$

So we get

$$\#\text{folds} = 1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21,\ 34, \ldots$$

This is the famous Fibonacci sequence, $F_n$.

# How fast does the taffy grow?

It is well-known that for large $n$,

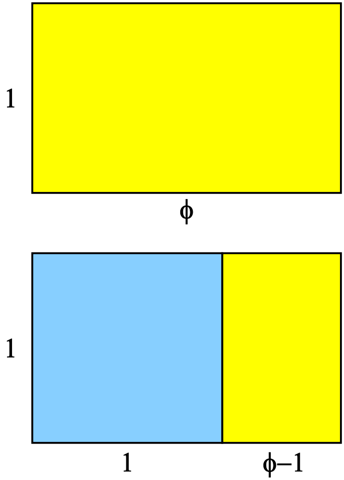
$$\frac{F_n}{F_{n-1}} \quad \rightarrow \quad \phi = \frac{1+\sqrt{5}}{2} = 1.6180\ldots$$

where $\phi$ is the Golden Ratio, also called the Golden Mean.

Along with $\pi$, $\phi$ is probably the best known number in mathematics. It seems to pop up everywhere...

So the ratio of lengths of the taffy between two successive steps is $\phi^2$, where the squared is due to the left/right alternation.

# The Golden Ratio, $\phi$




A rectangle has the proportions of the Golden Ratio if, after taking out a square, the remaining rectangle has the same proportions as the original:

$$\frac{\phi}{1} = \frac{1}{\phi - 1}$$

# A slightly more complex taffy puller

Now let's swap our rods twice each time.

[Matlab: `demo2`]

# Number of folds

We get for the number of left/right folds

$$\#\text{folds} = 1,\ 2,\ 5,\ 12,\ 29,\ 70,\ 169,\ 408\ldots$$

This sequence is given by

$$\#\text{folds}_n = 2\#\text{folds}_{n-1} + \#\text{folds}_{n-2}$$

For large $n$,

$$\frac{\#\text{folds}_n}{\#\text{folds}_{n-1}} \quad \to \quad \chi = 1 + \sqrt{2} = 2.4142\ldots$$

where $\chi$ is the Silver Ratio, a much less known number.

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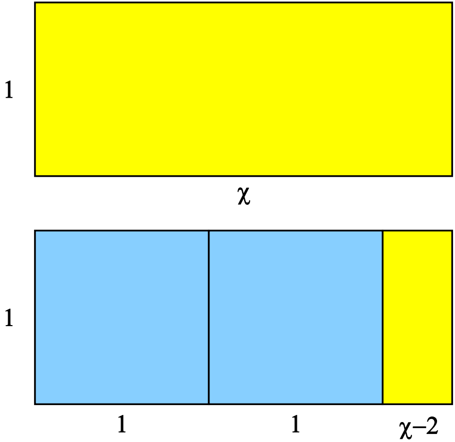
# The Silver Ratio, $\chi$

A rectangle has the proportions of the Silver Ratio if, after taking out two squares, the remaining rectangle has the same proportions as the original.



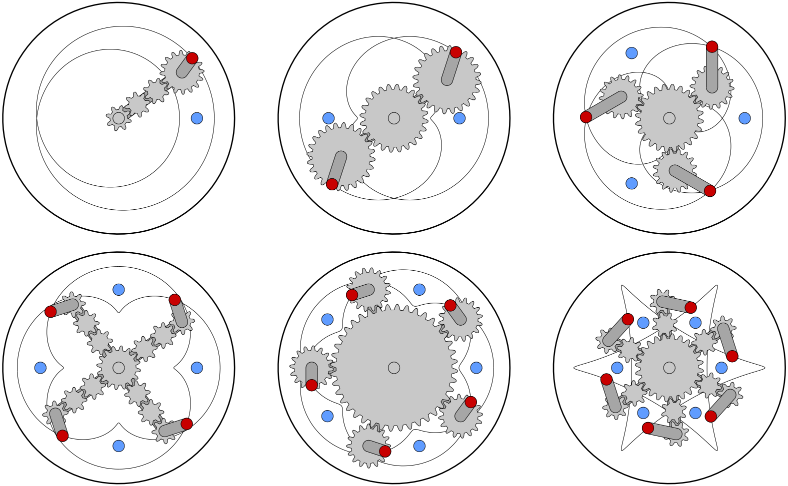

$$\frac{\chi}{1} = \frac{1}{\chi - 2}$$

# The original taffy puller

The taffy puller we originally presented stretches the taffy by $\chi^2$ at each 'period'.

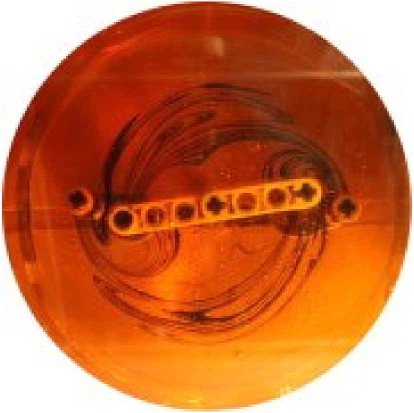
It's a special case of what we call Silver Mixers: devices that stretch by a power of the Silver Ratio.

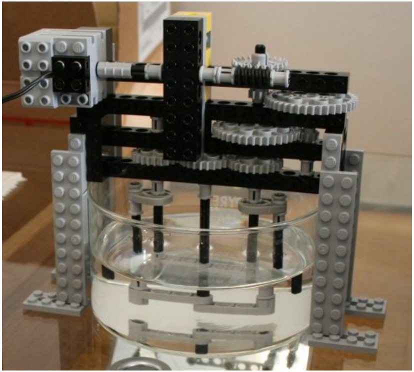
# Taffy superpullers!



[movie 5]

# Build it with Legos!





[movie 6] [movie 7] (Right-hand picture appearing on the cover of a math journal!)

# Some final thoughts

Is there a Bronze Ratio? Can we make such a taffy puller?

What about the taffy puller with four prongs?