

Simple models of stirring by swimming organisms

Jean-Luc Thiffeault¹ Steve Childress² Zhi George Lin³

¹Department of Mathematics
University of Wisconsin – Madison

²Courant Institute of Mathematical Sciences
New York University

³Institute for Mathematics and its Applications
University of Minnesota – Twin Cities

Complex Fluids and Flows, UBC, Vancouver
14 July 2011

Munk's Idea

Though it had been mentioned earlier, the first to seriously consider the role of ocean biomixing was Walter Munk (1966):

Abyssal recipes

WALTER H. MUNK*

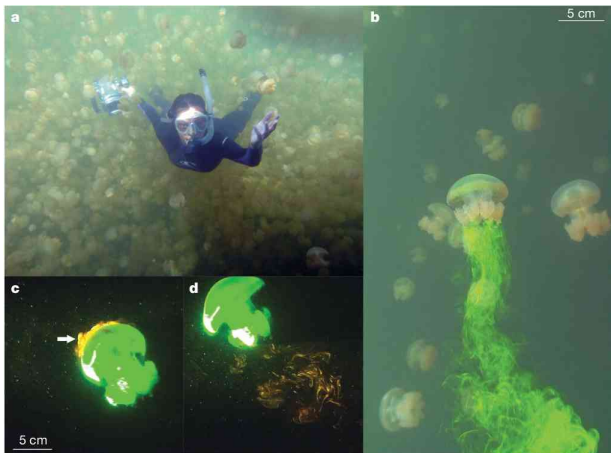
(Received 31 January 1966)

Abstract—Vertical distributions in the interior Pacific (excluding the top and bottom kilometer) are not inconsistent with a simple model involving a constant upward vertical velocity $w \approx 1.2 \text{ cm day}^{-1}$ and eddy diffusivity $\kappa \approx 1.3 \text{ cm}^2 \text{ sec}^{-1}$. Thus temperature and salinity can be fitted by exponential-like solutions to $[\kappa \cdot d^2/dz^2 - w \cdot d/dz] T, S = 0$, with $\kappa/w \approx 1 \text{ km}$ the appropriate "scale height." For Carbon 14 a decay term must be included, $[\quad]^{14}\text{C} = \mu^{14}\text{C}$; a fitting of the solution to the observed ^{14}C distribution yields $\kappa/w^2 \approx 200 \text{ years}$ for the appropriate "scale time," and permits w and

"... I have attempted, **without much success**, to interpret [the eddy diffusivity] from a variety of viewpoints: from mixing along the ocean boundaries, from thermodynamic and **biological processes**, and from internal tides."

In situ experiments

Katija & Dabiri (2009) looked at jellyfish:



[movie 1] (Palau's Jellyfish Lake.)

Displacement by a moving body

86

Mr. J. Clerk-Maxwell on

[Mar. 10,

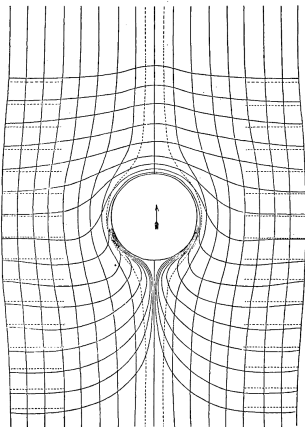


FIG. 1.

Fluid flowing past a fixed cylinder.

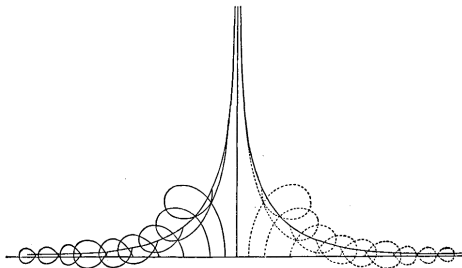


FIG. 2.

Paths of particles of the fluid when a cylinder moves through it.

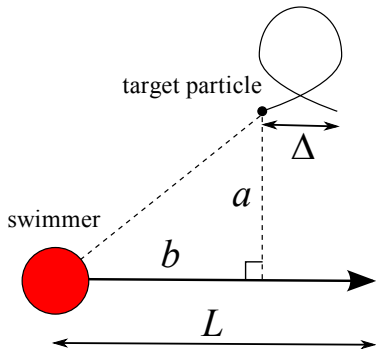
Maxwell (1869); Darwin (1953); Eames *et al.* (1994)

A sequence of kicks

Inspired by Einstein's theory of diffusion (Einstein, 1956): a test particle initially at $\mathbf{x}(0) = 0$ undergoes N encounters with an axially-symmetric swimming body:

$$\mathbf{x}(t) = \sum_{k=1}^N \Delta_L(a_k, b_k) \hat{\mathbf{r}}_k$$

$\Delta_L(a, b)$ is the displacement, a_k , b_k are **impact parameters**, and $\hat{\mathbf{r}}_k$ is a direction vector.



($a > 0$, but b can have either sign.)

Effective diffusivity

Putting this together,

$$\langle |\mathbf{x}|^2 \rangle = \frac{2Unt}{L} \int \Delta_L^2(a, b) da db = 4\kappa t, \quad \text{2D}$$

$$\langle |\mathbf{x}|^2 \rangle = \frac{2\pi Unt}{L} \int \Delta_L^2(a, b) a da db = 6\kappa t, \quad \text{3D}$$

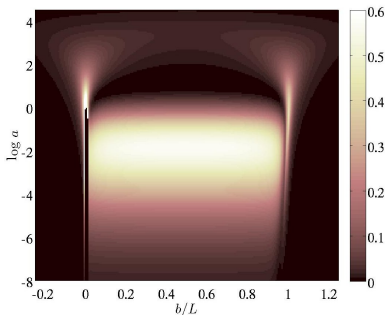
which defines the **effective diffusivity** κ .

If the number density is low ($nL^d \ll 1$), then encounters are rare and we can use this formula for a collection of particles.

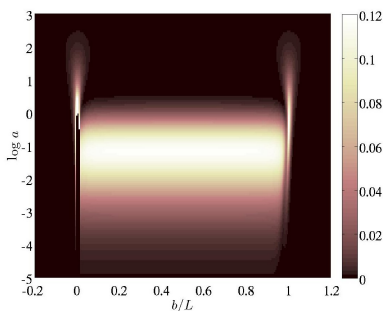
Inviscid cylinders and spheres (treadmill swimmer)

$$\kappa = \frac{\pi}{3} Un \int a^2 \Delta_L^2(a, b) d(\log a) d(b/L) \quad 3D$$

Notice $\Delta_L(a, b)$ is nonzero for $0 < b < L$; otherwise independent of b and L \implies have to cross point of closest approach.



$a^2 \Delta_L^2(a, b)$ (cylinder)

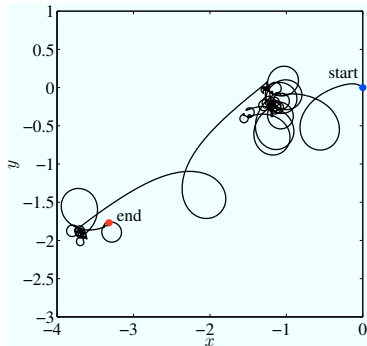
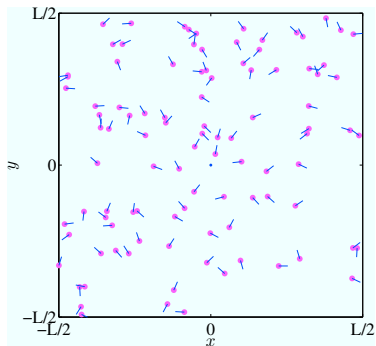


$a^2 \Delta_L^2(a, b)$ (sphere)

Numerical simulation

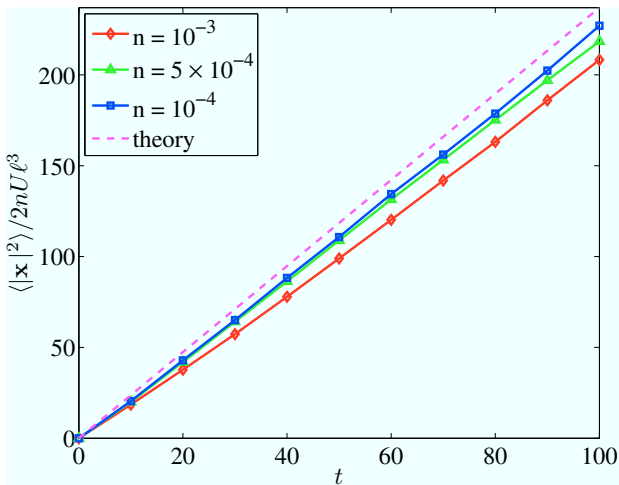
- Validate theory using simple simple simulations;
- Large periodic box;
- N_{swim} swimmers (cylinders of radius 1), initially at random positions, swimming in random direction with constant speed $U = 1$;
- Target particle initially at origin advected by the swimmers;
- Since dilute, superimpose velocities;
- Integrate for some time, compute $|\mathbf{x}(t)|^2$, repeat for a large number N_{real} of realizations, and average.

A 'gas' of swimmers



[movie 2] 100 cylinders, box size = 1000

How well does the dilute theory work?



Cloud of particles

t=10



t=630



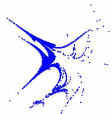
t=1255



t=1880



t=2505



t=3125



t=3750



t=4375

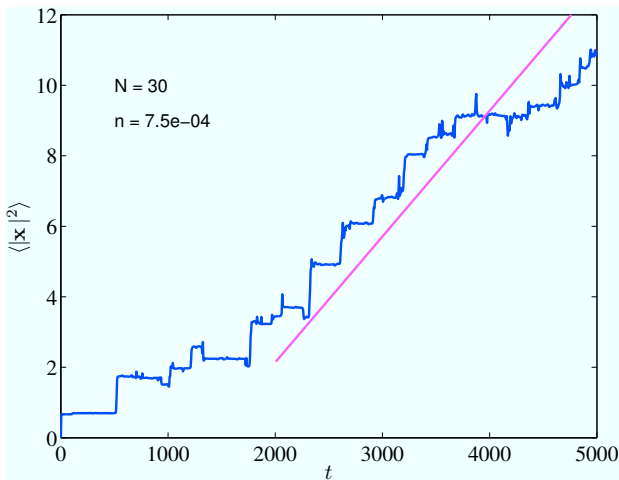


t=5000



[movie 3] (30 cylinders)

Cloud dispersion proceeds by steps

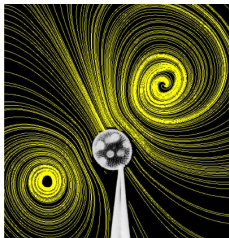


Squirmers

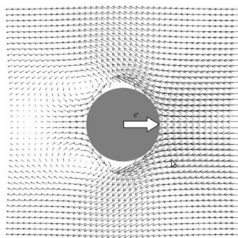
Considerable literature on transport due to microorganisms: Wu & Libchaber (2000); Hernandez-Ortiz *et al.* (2006); Saintillian & Shelley (2007); Ishikawa & Pedley (2007); Underhill *et al.* (2008); Ishikawa (2009); Leptos *et al.* (2009)

Lighthill (1952), Blake (1971), and more recently Ishikawa *et al.* (2006) have considered **squirmers**:

- Sphere in Stokes flow;
- Steady velocity specified at surface, to mimic cilia;
- Steady swimming condition imposed (no net force on fluid).



(Drescher *et al.*, 2009)



(Ishikawa *et al.*, 2006)

Typical squirmer

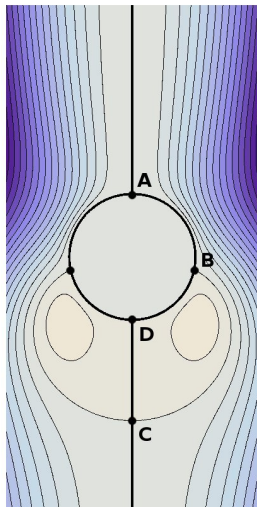
3D axisymmetric streamfunction for a typical squirmer, in cylindrical coordinates (ρ, z) :

$$\psi = -\frac{1}{2}\rho^2 + \frac{1}{2r^3}\rho^2 + \frac{3\beta}{4r^3}\rho^2 z \left(\frac{1}{r^2} - 1 \right)$$

where $r = \sqrt{\rho^2 + z^2}$, $U = 1$, radius of squirmer = 1.

β is the amplitude of the stresslet (distinguishes pushers/pullers).

We will use $\beta = 5$ for most of the remainder.

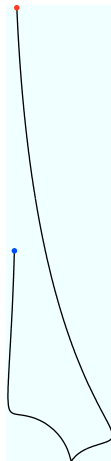


Particle motion for squirmer

A particle near the squirmer's swimming axis initially (blue) moves towards the squirmer.

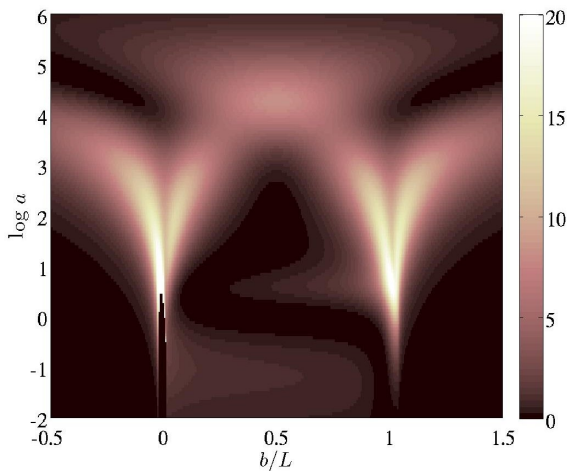
After the squirmer has passed the particle follows in the squirmer's wake.

(The squirmer moves from bottom to top.)

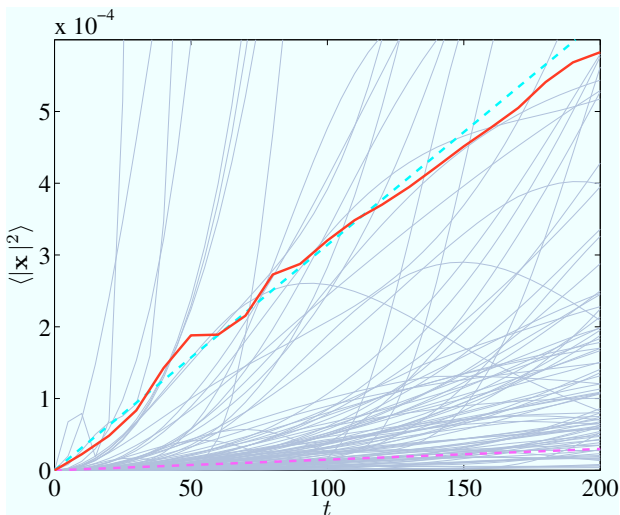


[movie 4]

Squirmer displacements $a^2 \Delta_L^2(a, b)$

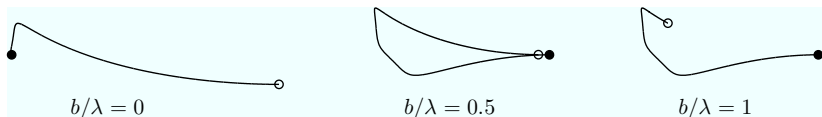


Squirmers: Transport



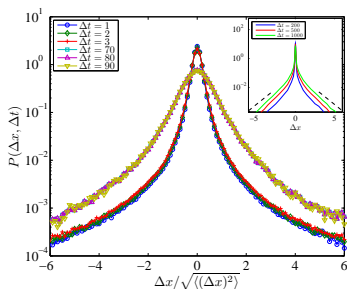
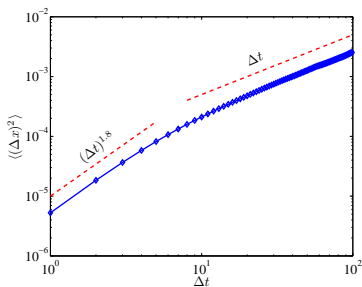
Squirmers: Trajectories

The two peaks in the displacement plot come from 'incomplete' trajectories:



For long path length, the effective diffusivity is **independent of the swimming path length**, and yet the dominant contribution arises from the finiteness of the path (**uncorrelated turning directions**).

Non-Gaussian PDFs of displacement



- Variance exhibits similar short-time anomalous scaling as in Wu & Libchaber (2000);
- PDF qualitatively matches experiments of Leptos *et al.* (2009). In our case, exponential tails are due to **sticking** at the stagnation points on the squirmer's body.

Conclusions

- Simple **dilute model** works well for a range of swimmers;
- Slip surfaces have an effective diffusivity that is **independent of path length**, for long path length;
- No-slip flows dominated by **sticking** and have a **log dependence** on path length;

Future work:

- Wake models and turbulence;
- PDF of scalar concentration;
- **Buoyancy effects** for the ocean case;
- Higher densities;
- Schooling: longer length scale?

This work was supported by the Division of Mathematical Sciences of the US National Science Foundation, under grants DMS-0806821 (J-LT) and DMS-0507615 (SC). ZGL is supported by NSF through the Institute for Mathematics and Applications.

- BLAKE, J. R. 1971 A spherical envelope approach to ciliary propulsion. *J. Fluid Mech.* **46**, 199–208.
- DARWIN, C. G. 1953 Note on hydrodynamics. *Proc. Camb. Phil. Soc.* **49** (2), 342–354.
- DEWAR, W. K., BINGHAM, R. J., IVERSON, R. L., NOWACEK, D. P., ST. LAURENT, L. C. & WIEBE, P. H. 2006 Does the marine biosphere mix the ocean? *J. Mar. Res.* **64**, 541–561.
- DOMBROWSKI, C., CISNEROS, L., CHATKAEW, S., GOLDSTEIN, R. E. & KESSLER, J. O. 2004 Self-concentration and large-scale coherence in bacterial dynamics. *Phys. Rev. Lett.* **93** (9), 098103.
- DRESCHER, K., LEPTOS, K., TUVAL, I., ISHIKAWA, T., PEDLEY, T. J. & GOLDSTEIN, R. E. 2009 Dancing *volvox*: hydrodynamic bound states of swimming algae. *Phys. Rev. Lett.* **102**, 168101.
- DRESCHER, K. D., GOLDSTEIN, R. E., MICHEL, N., POLIN, M. & TUVAL, I. 2010 Direct measurement of the flow field around swimming microorganisms. *Phys. Rev. Lett.* **105**, 168101.
- DUNKEL, J., PUTZ, V. B., ZAID, I. M. & YEOMANS, J. M. 2010 Swimmer-tracer scattering at low Reynolds number. *Soft Matter* **6**, 4268–4276.
- EAMES, I., BELCHER, S. E. & HUNT, J. C. R. 1994 Drift, partial drift, and Darwin's proposition. *J. Fluid Mech.* **275**, 201–223.
- EINSTEIN, A. 1956 *Investigations on the Theory of the Brownian Movement*. New York: Dover.
- GUASTO, J. S., JOHNSON, K. A. & GOLLUB, J. P. 2010 Oscillatory flows induced by microorganisms swimming in two-dimensions. *Phys. Rev. Lett.* **105**, 168102.
- HERNANDEZ-ORTIZ, J. P., DTOLZ, C. G. & GRAHAM, M. D. 2006 Transport and collective dynamics in suspensions of confined swimming particles. *Phys. Rev. Lett.* **95**, 204501.
- HUNTLEY, M. E. & ZHOU, M. 2004 Influence of animals on turbulence in the sea. *Mar. Ecol. Prog. Ser.* **273**, 65–79.
- ISHIKAWA, T. 2009 Suspension biomechanics of swimming microbes. *J. Roy. Soc. Interface* **6**, 815–834.
- ISHIKAWA, T. & PEDLEY, T. J. 2007 The rheology of a semi-dilute suspension of swimming model micro-organisms. *J. Fluid Mech.* **588**, 399–435.
- ISHIKAWA, T., SIMMONDS, M. P. & PEDLEY, T. J. 2006 Hydrodynamic interaction of two swimming model micro-organisms. *J. Fluid Mech.* **568**, 119–160.
- KATLA, K. & DABIRI, J. O. 2009 A viscosity-enhanced mechanism for biogenic ocean mixing. *Nature* **460**, 624–627.

- KUNZE, E., DOWER, J. F., BEVERIDGE, I., DEWEY, R. & BARTLETT, K. P. 2006 Observations of biologically generated turbulence in a coastal inlet. *Science* **313**, 1768–1770.
- LEPTOS, K. C., GUASTO, J. S., GOLLUB, J. P., PESCI, A. I. & GOLDSTEIN, R. E. 2009 Dynamics of enhanced tracer diffusion in suspensions of swimming eukaryotic microorganisms. *Phys. Rev. Lett.* **103**, 198103.
- LIGHTHILL, M. J. 1952 On the squirming motion of nearly spherical deformable bodies through liquids at very small Reynolds numbers. *Comm. Pure Appl. Math.* **5**, 109–118.
- LIN, Z., THIFFEAULT, J.-L. & CHILDRESS, S. 2011 Stirring by squirmers. *J. Fluid Mech.* **669**, 167–177.
- MAXWELL, J. C. 1869 On the displacement in a case of fluid motion. *Proc. London Math. Soc.* **s1-3** (1), 82–87.
- OSEEN, C. W. 1910 Über die Stokessche formel und über eine verwandte aufgabe in der hydrodynamik. *Ark. Mat. Astr. Fys.* **6** (29), 1–20.
- SAINTILLIAN, D. & SHELLEY, M. J. 2007 Orientational order and instabilities in suspensions of self-locomoting rods. *Phys. Rev. Lett.* **99**, 058102.
- THIFFEAULT, J.-L. & CHILDRESS, S. 2010 Stirring by swimming bodies. *Phys. Lett. A* **374**, 3487–3490.
- UNDERHILL, P. T., HERNANDEZ-ORTIZ, J. P. & GRAHAM, M. D. 2008 Diffusion and spatial correlations in suspensions of swimming particles. *Phys. Rev. Lett.* **100**, 248101.
- WU, X.-L. & LIBCHABER, A. 2000 Particle diffusion in a quasi-two-dimensional bacterial bath. *Phys. Rev. Lett.* **84**, 3017–3020.