

Turbulence and hagfish slime

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Joint work with **Randy Ewoldt** (UIUC)

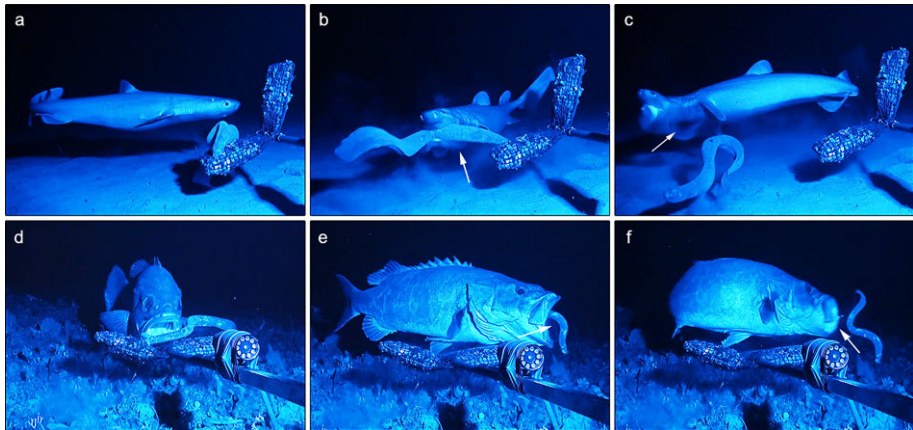
Workshop on Recent Advances in PDEs and Fluids, Stanford, CA
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- Not the prettiest fish.
- An ancient fish: no teeth.
- Only known living animal that has a skull but not a vertebral column.
- 77 species, average 50 cm.
- Eats worms as well as dead fish, by burrowing into their carcass.
They can feed through their own skin.





play movie

[Zintzen, V., Roberts, C. D., Anderson, M. J., Stewart, A. L., Struthers, C. D., & Harvey, E. S. (2011). *Scientific Reports*, 1, 131]



youtube movie

(see around 1 min mark)

slime in the lab: a promising material

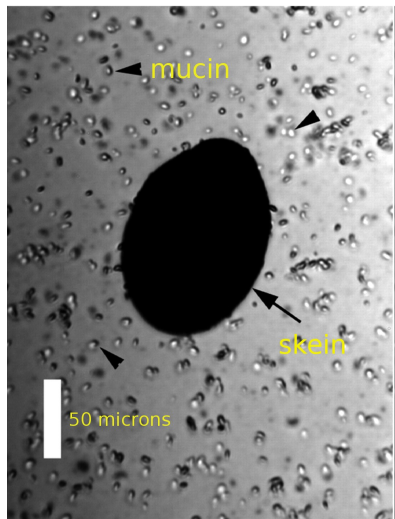


youtube movie

so what's inside the slime?



- .002% thread skein
- .0015% mucin
- 99.996% seawater (!)



[Fudge, D. S., Levy, N., Chiu, S., & Gosline, J. M. (2005). *J. Exp. Biol.* **208**, 4613–4625]

what's a skein?

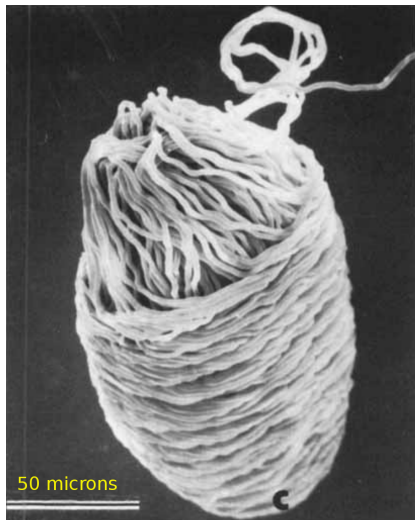


A **skein** consists of thread rolled into a ball.

Skeins are about 0.1 mm in size.

Thread length: **about 15 cm!**

The packing fraction is close to 1.



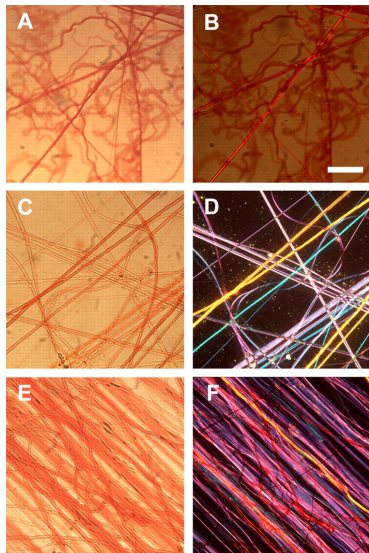
[Fernholm, B. (1981). *Acta Zool.* **62** (3), 137–145]

what happens when the skeins unravel?



The threads form a **network**, which gives the slime its properties.

The thread network determines the **rheology** of the slime.

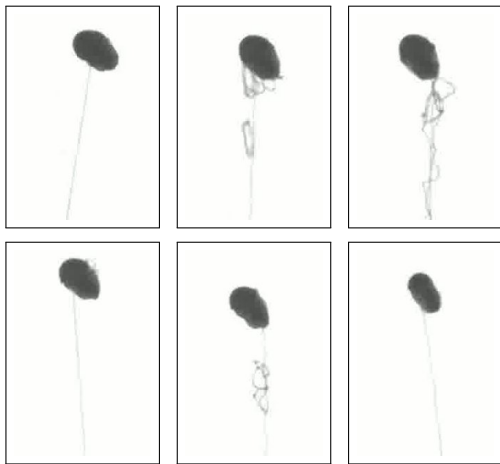


[Fudge *et al.* (2005)]

what happens when the skeins unravel?

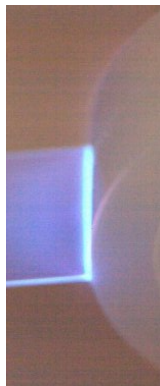
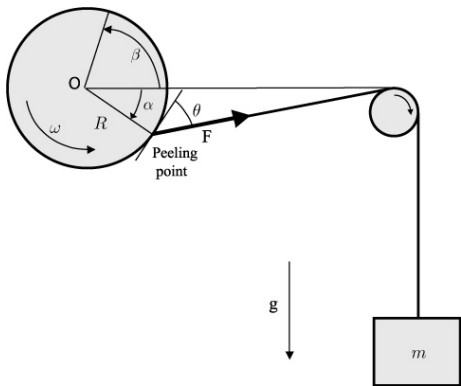


Here the skein is stuck to a glass slide:



play movie

[experiment by Randy Ewoltdt]



The dynamics at the peeling points can get very complicated and can even lead to **triboluminescence**.

[Figures from Cortet, P.-P., Ciccotti, M., & Vanel, L. (2007). *J. Stat. Mech.* **2007**, P03005 and Camara, C. G., Escobar, J. V., Hird, J. R., & Putterman, S. J. (2008). *Nature*, **455**, 1089–1092; see also Maugis & Barquins (1988); De *et al.* (2004).]



Work-energy theorem of Hong & Yue (1995):

$$\dot{U} = (T - F_0(V))V$$

- \dot{U} is the **rate of change in total energy** of the system;
- T is the **force** drawing out the thread;
- $F_0(V)$ is a velocity-dependent **peeling force**.

Neglect changes to the elastic energy of the tape ($\dot{U} = 0$):

$$T = F_0(V)$$



A simple model for the peeling force is

$$F_0(V) = \alpha V^m, \quad 0 \leq m \leq 1$$

which we solve for the peeling velocity:

$$V = (T/\alpha)^{1/m}$$

The total length $L(t)$ of thread drawn out thus satisfies

$$\dot{L} = (T/\alpha)^{1/m}$$



Relate R , the **skein radius**, and L using mass conservation:

$$\frac{d}{dt} \left(\frac{4}{3} \pi \eta R^3 + \pi r^2 L \right) = 0 \quad \implies \quad \dot{L} = -4\eta R^2 \dot{R} / r^2,$$

where r is the **thread radius** and $\eta \leq 1$ is the **packing fraction** of thread into the spherical skein.

Under constant tension T , we can easily solve for the **depletion time**

$$t_{\text{dep}} = \frac{4\eta R_0^3}{3r^2} (T/\alpha)^{1/m}$$

to run out of thread, given an initial skein radius R_0 .

So far there is no fluid.

To model the effect of the fluid on the thread, we use **resistive force theory**:

$$8\pi\mu\delta(\mathbf{x}_t - \mathbf{u}) = (1 + 2\delta)T\mathbf{x}_{ss} + 2T_s\mathbf{x}_s \quad (\text{force balance})$$

$$2T_{ss} - (1 + 2\delta)T|\mathbf{x}_{ss}|^2 = -8\pi\mu\delta\mathbf{x}_s \cdot \mathbf{u}_s \quad (\text{torque balance})$$

where $\mathbf{x}(s, t)$ is the **Eulerian** position of a thread segment as a function of the **Lagrangian** label s (arc length).

$\delta = -1/\log(\varepsilon^2 e)$ is the **slenderness parameter**, $\varepsilon = r/L$ is the **slenderness ratio**, with r the thread radius and L its length.

(**Resistive force theory** differs from **slender body theory** in the neglect of the **nonlocal term**, which is appropriate when the thread is fairly straight.)



Assume the skein is immersed a simple 1D flow

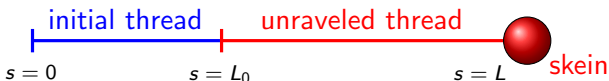
$$\mathbf{u}(x, y, t) = u(x, t) \hat{\mathbf{x}}$$

at the origin. The thread remains straight and aligned with the horizontal.

Resistive force theory for a straight filament then says

$$\begin{aligned} 8\pi\mu\delta(\mathbf{x}_t - \mathbf{u}) &= 2T_s\mathbf{x}_s \\ 2T_{ss} &= -8\pi\mu\delta\mathbf{x}_s \cdot \mathbf{u}_s \end{aligned}$$

Appropriate boundary conditions have to be imposed at the ends of the thread.



- The Lagrangian arc length parameter values $s \in [0, L_0]$ correspond to the 'initial' piece of thread.
- The thread added by unraveling is $s \in (L_0, L(t)]$.

Let's start with a skein that is held in place at \mathbf{X} , with thread unraveling due to hydrodynamics. The boundary conditions are then

$$T(0, t) = 0, \quad (\text{free end}); \quad \mathbf{x}(L(t), t) = \mathbf{X}, \quad (\text{tethered end}).$$

We need one more boundary condition on T , which is obtained by taking the force balance equation, evaluating it at $s = L(t)$, and using

$$\frac{d}{dt}\mathbf{x}(s = L, t) = \mathbf{x}_t + \mathbf{x}_s \dot{L} = \frac{d}{dt}\mathbf{X} = 0$$

We obtain

$$T_s = -4\pi\mu\delta(\dot{L} + \mathbf{x}_s \cdot \mathbf{u}) \quad \text{at} \quad s = L.$$

We skip some details of the derivation. Once we have the force at the tether point we can use it in our peeling law $F = \alpha V^m$.

In the end we get the equation

$$(\dot{L})^m = -\frac{4\pi\mu\delta}{\alpha} L (\bar{u}_X(L) + \dot{L}).$$

where

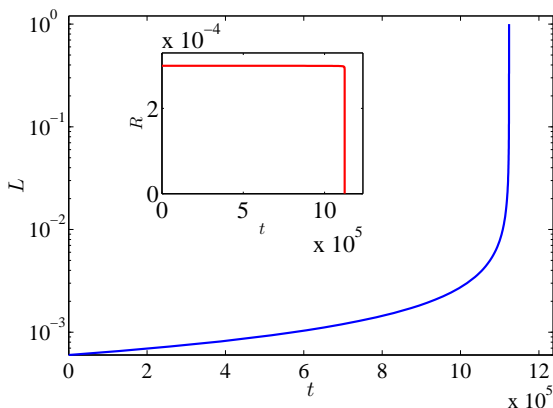
$$\bar{u}_X(L, t) := \frac{1}{L} \int_{X-L}^X u(x, t) dx$$

is the thread-averaged velocity.

Not surprisingly, the force is determined by the average of the velocity on the filament.

This nonlinear ODE cannot be solved analytically, except in some asymptotic limits for special choices of m .

Typical simulation for an extensional flow $u(x, t) = \lambda(t)x$:



The skein unravels suddenly once it gets long enough.



A more relevant situation is to have the skein-thread system free to move.

The tension at $s = L(t)$ — the end unraveling from the skein — is equal to the **drag force on a sphere** of radius R :

$$T = 6\pi\mu R \mathbf{x}_s \cdot (\mathbf{u} - \mathbf{x}_t), \quad s = L(t).$$

The other end is **free**:

$$T = 0, \quad s = 0.$$



The Eulerian position of a thread element is

$$x = X(t) - L(t) + s$$

where $X(t)$ is the **position of the skein**.

Hence, using the tension in our peeling law:

$$(\dot{L})^m = 6\pi\mu R\alpha^{-1}(\dot{L} - \dot{X} + u(X, t))$$

This is not closed: we need to find a separate equation for \dot{X} . We do this by solving the equations of resistive force theory.

Skipping some algebraic details, we eventually obtain the system of differential equations

$$\begin{aligned}(\dot{L})^m &= 6\pi\mu\alpha^{-1}R\bar{u}(X, L, t) \\ \dot{X} &= \dot{L} + u(X, t) - \bar{u}(X, L, t)\end{aligned}$$

where

$$\bar{u}(X, L, t) = \frac{1}{L + (3R/2\delta)} \int_{X-L}^X \{u(X, t) - u(x, t)\} dx$$

Note that R and L are related by [mass conservation](#).



For an extensional flow

$$u(x, t) = \lambda(t)x$$

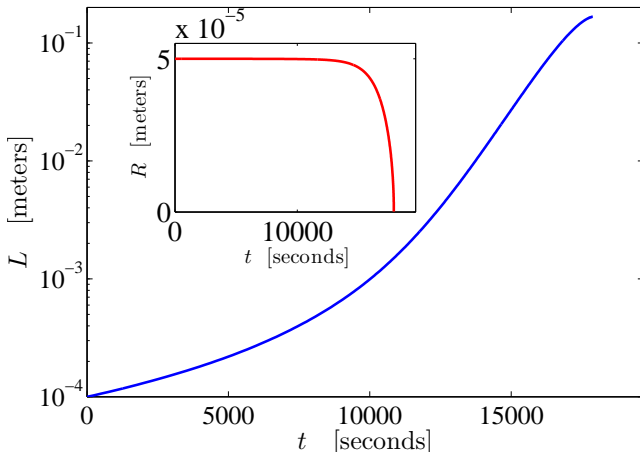
we can reduce the system to one ODE:

$$\boxed{(\dot{L})^m = 3\pi\mu\alpha^{-1}\lambda RL^2/(L + (3R/2\delta))}$$

where again $R = R(L)$, and also $\delta = \delta(L)$ (slenderness parameter).

[Recall: $\delta = -1/\log(\varepsilon^2 e)$ with $\varepsilon = r/L$ the slenderness ratio.]

Again, this cannot be solved analytically, except in some asymptotic limits for special choices of m .



Numerical solution for $r = 1 \mu\text{m}$, $R_0 = 50 \mu\text{m}$, $L_0 = 2R_0$, $\mu = 1.5 \times 10^{-3} \text{ Pa s}$,
 $m = 1/3$, $\alpha = 8 \times 10^{-4} \text{ N (m/s)}^{-1/3}$, $\eta = 1$, $\lambda = 1 \text{ s}^{-1}$.



- The **depletion time** is about 10^3 s.
- Problem: this is 3 orders of magnitude too long!
- Possible issues:
 - No real idea what peeling force parameter values to use. (Here used educated guess based on tape.) Changing these can radically alter the results.
 - Maybe adjust the drag force if the filament doesn't remain straight: go beyond resistive force to full slender-body theory.
 - Are the **mucins** important? Experiments suggest so but their role is unclear. They might catalyze the peeling somehow, or stick to the filament and increase the drag force.
 - **Proper rheological experiments needed** (Randy Ewoldt)!
 - Use a more 'mixing' flow, closer to **turbulence**.
- Future research: **network created by threads**.

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