Unraveling hagfish slime

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WISCONSIN
UNIVERSITY OF WISCONSIN–MADISON

- Not the prettiest fish.
- An ancient fish: no teeth.
- Only known living animal that has a skull but not a vertebral column.
- 77 species, average 50 cm.
- Eats worms as well as dead fish, by burrowing into their carcass. They can feed through their own skin.
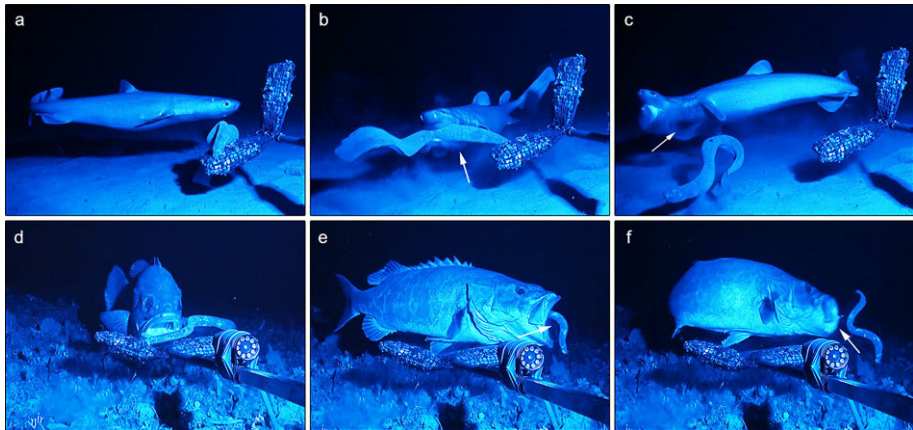
play movie

[Zintzen, V., Roberts, C. D., Anderson, M. J., Stewart, A. L., Struthers, C. D., & Harvey, E. S. (2011). *Scientific Reports,* **1**, 131]

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youtube movie (see around 1 min mark)

youtube movie

['Slime eels' explode on highway after bizarre traffic accident]

- .002% thread skein
- .0015% mucin
- 99.996% seawater (!)
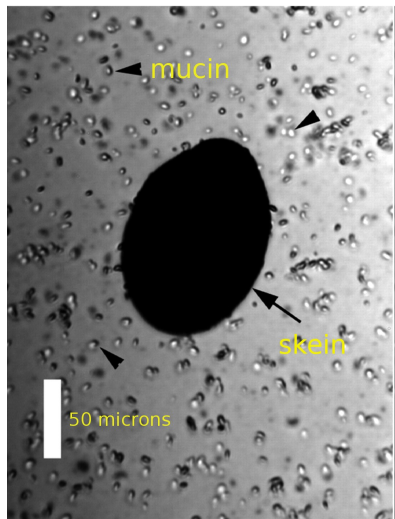
[Fudge, D. S., Levy, N., Chiu, S., & Gosline, J. M. (2005). *J. Exp. Biol.* **208**, 4613–4625]

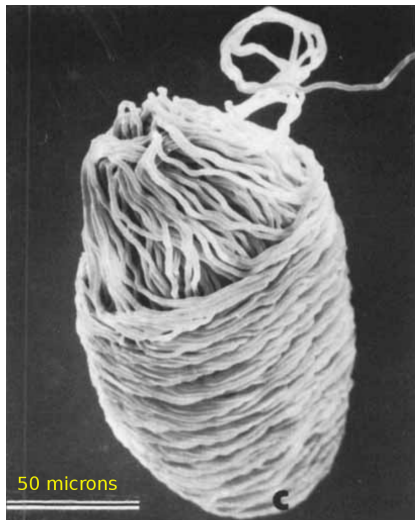A skein consists of thread rolled into a ball.

Skeins are about 0.1 mm in size.

Thread length: about 15 cm!

The packing fraction is close to 1.

50 microns

[Fernholm, B. (1981). *Acta Zool.* **62** (3), 137–145]

The threads form a network, which gives the slime its properties.

The thread network determines the rheology of the slime.
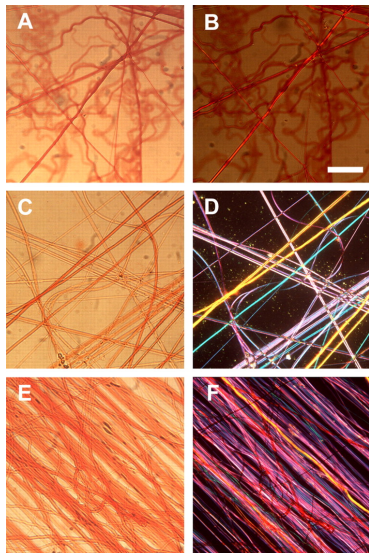
[Fudge *et al.* (2005)]

Here the skein is stuck to a glass slide:
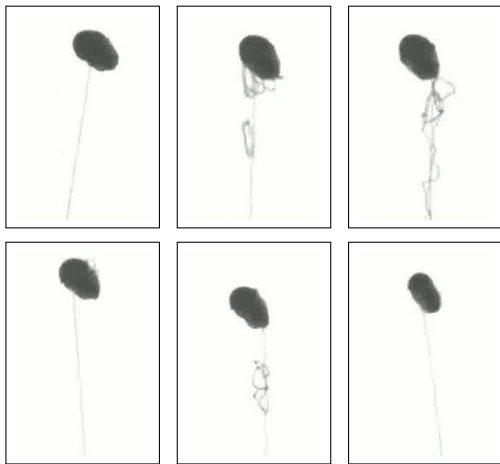
[experiment by Randy Ewoldt]
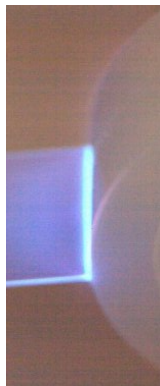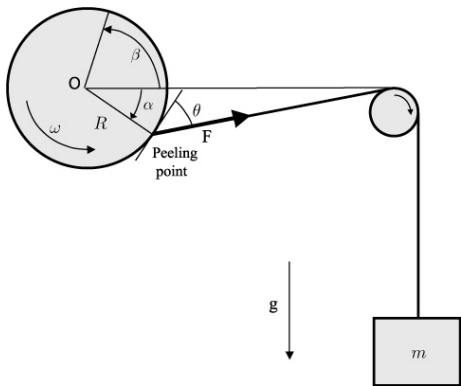
The dynamics at the peeling points can get very complicated and can even lead to triboluminescence.

[Figures from Cortet, P.-P., Ciccotti, M., & Vanel, L. (2007). *J. Stat. Mech.* **2007**, P03005 and Camara, C. G., Escobar, J. V., Hird, J. R., & Putterman, S. J. (2008). *Nature,* **455**, 1089–1092; see also Maugis & Barquins (1988); De *et al.* (2004).]

Work-energy theorem of Hong & Yue (1995):

$$\dot{U} = (T - F_0(V))V$$

- $\dot{U}$ is the rate of change in total energy of the system;
- T is the force drawing out the thread;
- $F_0(V)$ is a velocity-dependent peeling force.

Neglect changes to the elastic energy of the tape ($\dot{U} = 0$):

$$\boxed{T = F_0(V)}$$

A simple model for the peeling force is

$$F_0(V) = \alpha V^m, \qquad 0 \le m \le 1$$

which we solve for the peeling velocity:

$$V = (T/\alpha)^{1/m}$$

The total length $L(t)$ of thread drawn out thus satisfies

$$\boxed{\dot{L} = (T/\alpha)^{1/m}}$$

Relate R, the skein radius, and L using mass conservation:

$$\frac{d}{dt}\left(\tfrac{4}{3}\pi\eta R^3 + \pi r^2 L\right) = 0 \quad \implies \quad \dot{L} = -4\eta R^2 \dot{R}/r^2,$$

where r is the thread radius and $\eta \leq 1$ is the packing fraction of thread into the spherical skein.

Under constant tension T, we can easily solve for the depletion time

$$t_{\text{dep}} = \frac{4\eta R_0^3}{3r^2}\left(T/\alpha\right)^{1/m}$$

to run out of thread, given an initial skein radius R_0.

So far there is no fluid.

To model the effect of the fluid on the thread, we use resistive force theory:

$$8\pi\mu\delta\left(\mathbf{x}_t - \mathbf{u}\right) = (1 + 2\delta)\,T\mathbf{x}_{ss} + 2T_s\mathbf{x}_s \qquad \text{(force balance)}$$

$$2T_{ss} - (1 + 2\delta)\,T|\mathbf{x}_{ss}|^2 = -8\pi\mu\delta\mathbf{x}_s \cdot \mathbf{u}_s \qquad \text{(torque balance)}$$

where $\mathbf{x}(s, t)$ is the Eulerian position of a thread segment as a function of the Lagrangian label s (arc length).

$\delta = -1/\log(\varepsilon^2 e)$ is the slenderness parameter, $\varepsilon = r/L$ is the slenderness ratio, with r the thread radius and L its length.

(Resistive force theory differs from slender body theory in the neglect of the nonlocal term, which is appropriate when the thread is fairly straight.)

Assume the skein is immersed a simple 1D flow

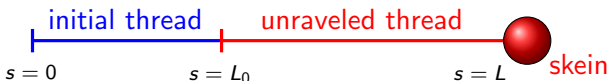$$\mathbf{u}(x, y, t) = u(x, t)\,\hat{\mathbf{x}}$$

at the origin. The thread remains straight and aligned with the horizontal.

Resistive force theory for a straight filament then says

$$8\pi\mu\delta\left(\mathbf{x}_t - \mathbf{u}\right) = 2T_s\mathbf{x}_s$$
$$2T_{ss} = -8\pi\mu\delta\mathbf{x}_s \cdot \mathbf{u}_s$$

Appropriate boundary conditions have to be imposed at the ends of the thread.

initial thread — unraveled thread — skein

$s = 0$ $s = L_0$ $s = L$

- The Lagrangian arc length parameter values $s \in [0, L_0]$ correspond to the 'initial' piece of thread.
- The thread added by unraveling is $s \in (L_0, L(t)]$.

Let's start with a skein that is held in place at $\mathbf{X}$, with thread unraveling due to hydrodynamics. The boundary conditions are then

$$T(0, t) = 0, \quad \text{(free end);} \qquad \mathbf{x}(L(t), t) = \mathbf{X}, \quad \text{(tethered end).}$$

We need one more boundary condition on T, which is obtained by taking the force balance equation, evaluating it at $s = L(t)$, and using

$$\frac{d}{dt}\mathbf{x}(s = L, t) = \mathbf{x}_t + \mathbf{x}_s \dot{L} = \frac{d}{dt}\mathbf{X} = 0$$

We obtain

$$T_s = -4\pi\mu\delta\left(\dot{L} + \mathbf{x}_s \cdot \mathbf{u}\right) \quad \text{at} \quad s = L.$$

tethered skein equation

We skip some details of the derivation. Once we have the force at the tether point we can use it in our peeling law $F = \alpha V^m$.

In the end we get the equation

$$(\dot{L})^m = -\frac{4\pi\mu\delta}{\alpha} L \left(\bar{u}_X(L) + \dot{L}\right).$$
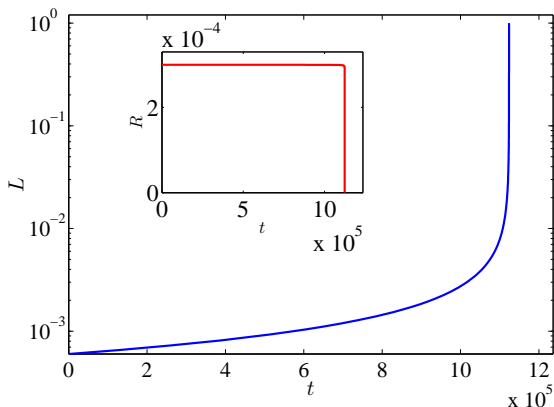
where

$$\bar{u}_X(L, t) := \frac{1}{L} \int_{X-L}^{X} u(x, t)\, \mathrm{d}x$$

is the thread-averaged velocity.

Not surprisingly, the force is determined by the average of the velocity on the filament.

This nonlinear ODE cannot be solved analytically, except in some asymptotic limits for special choices of m.

Typical simulation for an extensional flow $u(x, t) = \lambda(t)x$:

The skein unravels suddenly once it gets long enough.

A more relevant situation is to have the skein-thread system free to move.

The tension at $s = L(t)$ — the end unraveling from the skein — is equal to the drag force on a sphere of radius R:

$$T = 6\pi\mu R\, \mathbf{x}_s \cdot (\mathbf{u} - \mathbf{x}_t), \qquad s = L(t).$$

The other end is free:

$$T = 0, \qquad s = 0.$$

The Eulerian position of a thread element is

$$x = X(t) - L(t) + s$$

where $X(t)$ is the position of the skein.

Hence, using the tension in our peeling law:

$$(\dot{L})^m = 6\pi\mu R\alpha^{-1}(\dot{L} - \dot{X} + u(X, t))$$

This is not closed: we need to find a separate equation for $\dot{X}$. We do this by solving the equations of resistive force theory.

Skipping some algebraic details, we eventually obtain the system of differential equations

$$(\dot{L})^m = 6\pi\mu\alpha^{-1}R\,\bar{u}(X, L, t)$$
$$\dot{X} = \dot{L} + u(X, t) - \bar{u}(X, L, t)$$

where

$$\bar{u}(X, L, t) = \frac{1}{L + (3R/2\delta)} \int_{X-L}^{X} \{u(X, t) - u(x, t)\}\,\mathrm{d}x$$

Note that R and L are related by mass conservation.

For an extensional flow
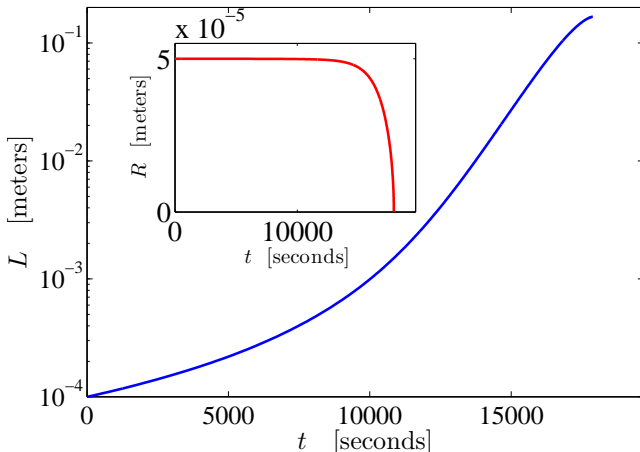
$$u(x, t) = \lambda(t)x$$

we can reduce the system to one ODE:

$$\boxed{(\dot{L})^m = 3\pi\mu\alpha^{-1}\lambda\, RL^2/(L + (3R/2\delta))}$$

where again $R = R(L)$, and also $\delta = \delta(L)$ (slenderness parameter).

[Recall: $\delta = -1/\log(\varepsilon^2 \mathrm{e})$ with $\varepsilon = r/L$ the slenderness ratio.]

Again, this cannot be solved analytically, except in some asymptotic limits for special choices of m.

Numerical solution for $r = 1\,\mu$m, $R_0 = 50\,\mu$m, $L_0 = 2R_0$, $\mu = 1.5 \times 10^{-3}\,\mathrm{Pa\,s}$, $m = 1/3$, $\alpha = 8 \times 10^{-4}\,\mathrm{N\,(m/s)^{-1/3}}$, $\eta = 1$, $\lambda = 1\,\mathrm{s^{-1}}$.

- The depletion time is about $10^3\,\mathrm{s}$.
- Problem: this is 3 orders of magnitude too long!
- Possible issues:
  - No real idea what peeling force parameter values to use. (Here used educated guess based on tape.) Changing these can radically alter the results.
  - Maybe adjust the drag force if the filament doesn't remain straight: go beyond resistive force to full slender-body theory.
  - Are the mucins important? Experiments suggest so but their role is unclear. They might catalyze the peeling somehow, or stick to the filament and increase the drag force.
  - Proper rheological experiments needed (Randy Ewoldt)!
  - Use a more 'mixing' flow, closer to turbulence.
- Future research: network created by threads.

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