

# slimy math

## modeling hagfish slime

Jean-Luc Thiffeault

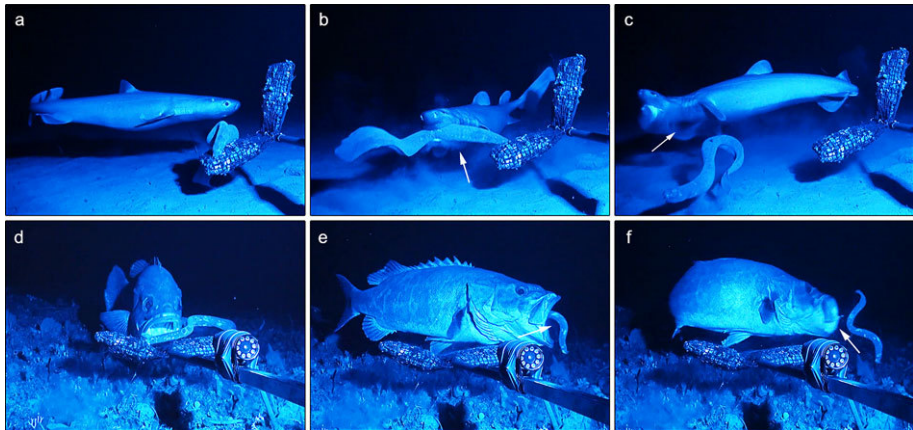
Department of Mathematics  
University of Wisconsin – Madison

Why math matters  
University of Wisconsin – Madison  
9 November 2018



- Not the prettiest fish.
- An ancient fish: no teeth.
- Only known living animal that has a skull but not a vertebral column.
- 77 species, average 50 cm.
- Eats worms as well as dead fish, by burrowing into their carcass.  
They can feed through their own skin.





play movie

[Zintzen, V., Roberts, C. D., Anderson, M. J., Stewart, A. L., Struthers, C. D., & Harvey, E. S. (2011). *Scientific Reports*, 1, 131]



youtube movie

(see around 1 min mark)

# slime in the lab: a promising material

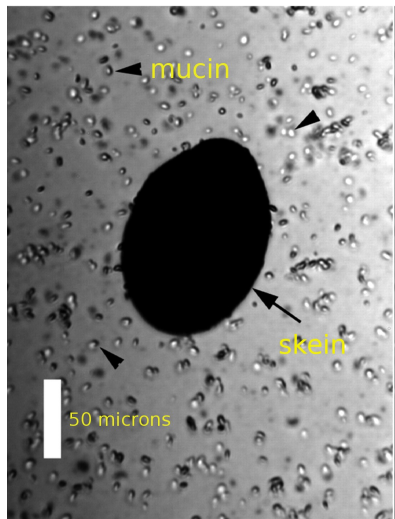


youtube movie

# so what's inside the slime?



- .002% thread skein
- .0015% mucin
- 99.996% seawater (!)



[Fudge, D. S., Levy, N., Chiu, S., & Gosline, J. M. (2005). *Journal of Experimental Biology*, **208**, 4613–4625]

# what's a skein?

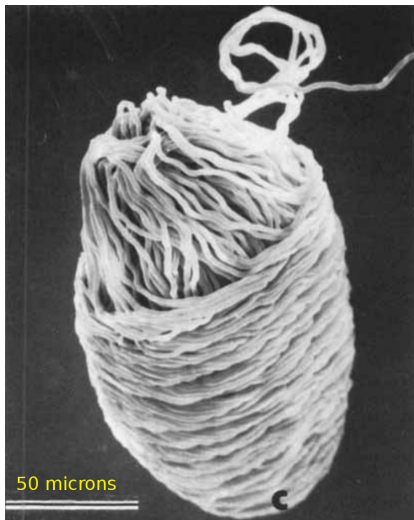


A **skein** consists of thread rolled into a ball.

Skeins are about 0.1 mm in size.

Thread length: **about 15 cm!**

The packing fraction is close to 1.



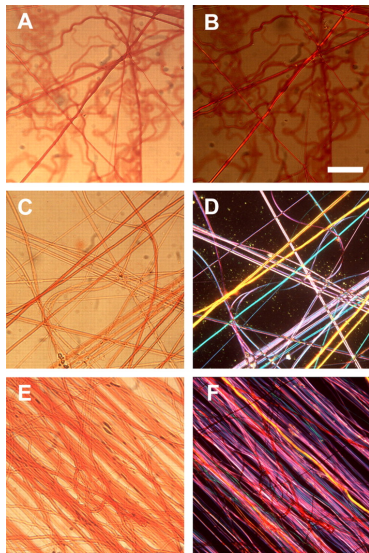
[Fernholm, B. (1981). *Acta Zoologica*, **62** (3), 137–145]

# what happens when the skeins unravel?



The threads form a **network**, which gives the slime its properties.

The thread network determines the **rheology** of the slime.



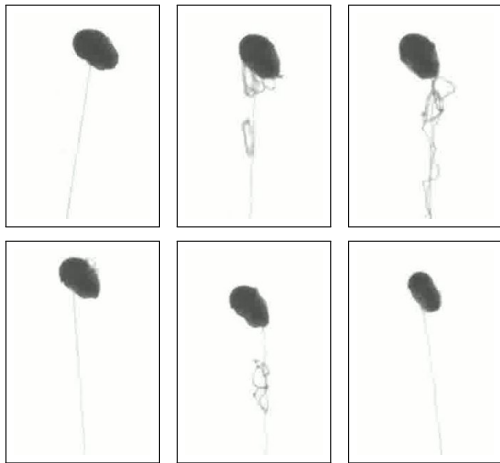
[Fudge *et al.* (2005)]



# what happens when the skeins unravel?



Here the skein is stuck to a glass slide:



play movie

[experiment by Randy Ewoltdt]



A simple model for the peeling force is

$$F_0(V) = \alpha V^m, \quad 0 \leq m \leq 1$$

which we solve for the peeling velocity:

$$V = (T/\alpha)^{1/m}$$

The total length  $L(t)$  of thread drawn out thus satisfies

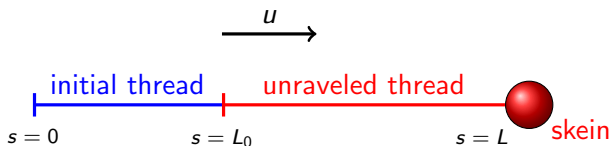
$$\dot{L} = (T/\alpha)^{1/m}$$



Relate  $R$ , the **skain radius**, and  $L$  using mass conservation:

$$\frac{d}{dt} \left( \frac{4}{3}\pi\eta R^3 + \pi r^2 L \right) = 0 \quad \implies \quad \dot{L} = -4\eta R^2 \dot{R} / r^2,$$

where  $r$  is the **thread radius** and  $\eta \leq 1$  is the **packing fraction** of thread into the spherical skain.



$$(\dot{L})^m = 6\pi\mu\alpha^{-1}R(L)(u(L, t) - \dot{L}). \quad (2)$$

This is a **Differential Algebraic Equation** (DAE), a generalization of ODEs where we cannot solve for  $\dot{L}$  explicitly.

We solve this for many different cases in our recent paper  
 G. Chaudhary, R. Ewoldt, and J.-L.T., "Unraveling hagfish slime."  
<http://arxiv.org/abs/1809.05914>

be careful about transporting hagfish in your car



[‘Slime eels’ explode on highway after bizarre traffic accident]

be careful about transporting hagfish in your car



- Camara, C. G., Escobar, J. V., Hird, J. R., & Putterman, S. J. (2008). *Nature*, **455**, 1089–1092.
- Cortet, P.-P., Ciccotti, M., & Vanel, L. (2007). *Journal of Statistical Mechanics*, **2007**, P03005.
- De, R., Maybhat, A., & Ananthakrishna, G. (2004). *Physical Review E*, **70**, 046223.
- Downing, S. W., Salo, W. L., Spitzer, R. H., & Koch, E. A. (1981). *Science*, **214**, 1143–1145.
- Downing, S. W., Spitzer, R. H., Koch, E. A., & Salo, W. L. (1984). *Journal of Cell Biology*, **98** (2), 653–669.
- Downing, S. W., Spitzer, R. H., Salo, W. L., Downing, S. D., Saidel, L. J., & Koch, E. A. (1981). *Science*, **212**, 326–327.
- Fernholm, B. (1981). *Acta Zoologica*, **62** (3), 137–145.
- Fudge, D. S., Levy, N., Chiu, S., & Gosline, J. M. (2005). *Journal of Experimental Biology*, **208**, 4613–4625.
- Hinch, E. J. (1976). *Journal of Fluid Mechanics*, **74** (2), 317–333.
- Hong, D. C. & Yue, S. (1995). *Physical Review Letters*, **74** (2), 254–257.
- Maugis, D. & Barquins, M. (1988). In: *Adhesion 12*, (Allen, K. W., ed) p. 205, London: Elsevier.
- Powers, T. R. (2010). *Reviews of Modern Physics*, **82**, 1607–1631.
- Spitzer, R. H., Downing, S. W., Koch, E. A., Salo, W. L., & Saidel, L. J. (1984). *Journal of Cell Biology*, **98** (2), 670–677.
- Tornberg, A.-K. & Shelley, M. J. (2004). *Journal of Computational Physics*, **196**, 8–40.
- Young, Y.-N. (2010). *Physical Review E*, **82**, 016309.
- Zintzen, V., Roberts, C. D., Anderson, M. J., Stewart, A. L., Struthers, C. D., & Harvey, E. S. (2011). *Scientific Reports*, **1**, 131.