the mathematics of taffy pullers

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the taffy puller

Taffy is a type of candy.

Needs to be pulled: this aerates it and makes it lighter and chewier.

We can assign a growth: length multiplier per period.

[movie by M. D. Finn]

play movie

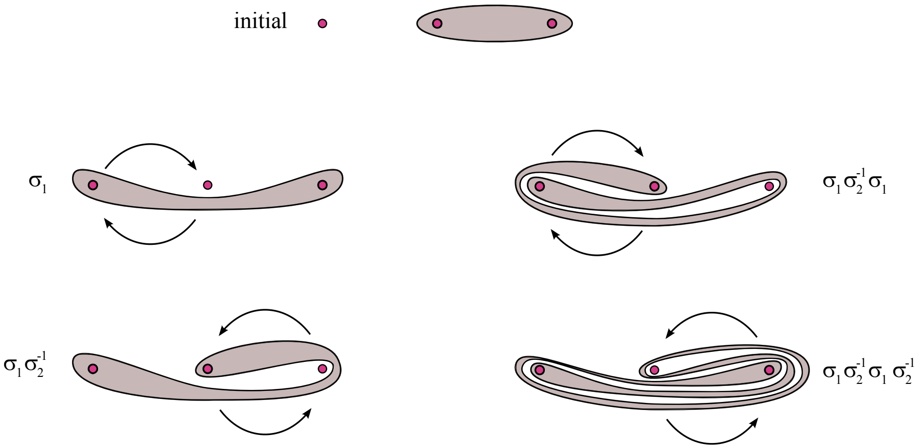
four-pronged taffy puller

play movie

http://www.youtube.com/watch?v=Y7tlHDsquVM

[MacKay (2001); Halbert & Yorke (2014)]

a simple taffy puller



Count alternating left/right folds. 1, 1, 2, 3, 5, 8, 13, 21, 34, . . .

number of folds

Let's count alternating left/right folds. The sequence is

$$\#\text{folds} = 1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21,\ 34, \dots$$

What is the rule?

$$\#\text{folds}_n = \#\text{folds}_{n-1} + \#\text{folds}_{n-2}$$

This is the famous Fibonacci sequence, F_n.

how fast does the taffy grow?

It is well-known that for large n,

$$\frac{F_n}{F_{n-1}} \quad \to \quad \phi = \frac{1+\sqrt{5}}{2} = 1.6180\ldots$$

where ϕ is the Golden Ratio, also called the Golden Mean.

So the ratio of lengths of the taffy between two successive steps is ϕ^2, where the squared is due to the left/right alternation.

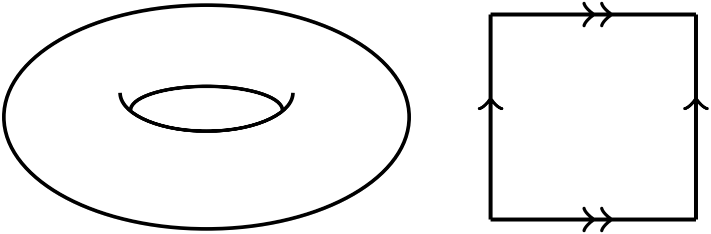
Hence, the growth factor for this taffy puller is

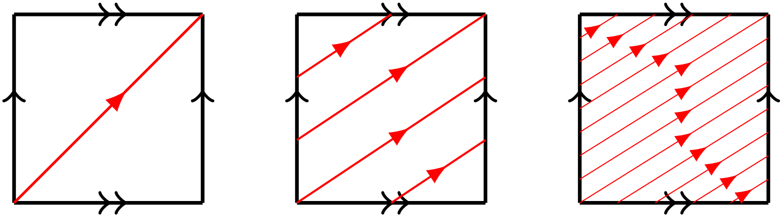
$$\boxed{\phi^2 = \phi + 1 = 2.6180\ldots.}$$

The standard taffy pullers have the lesser-known Silver Ratio $(1+\sqrt{2})$ as their growth factor.

maps on the torus (donut)

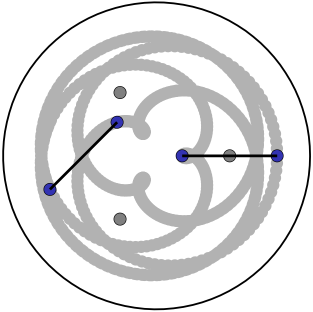
There is a deep mathematical connection between taffy pullers and transformations (maps) of the torus:

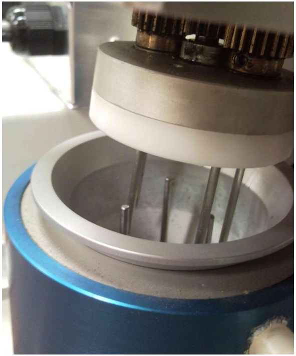


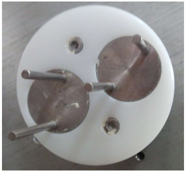


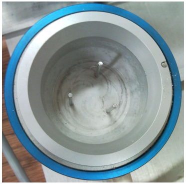
the mixograph

A modern 'taffy puller' is the mixograph, a device for measuring the properties of dough:



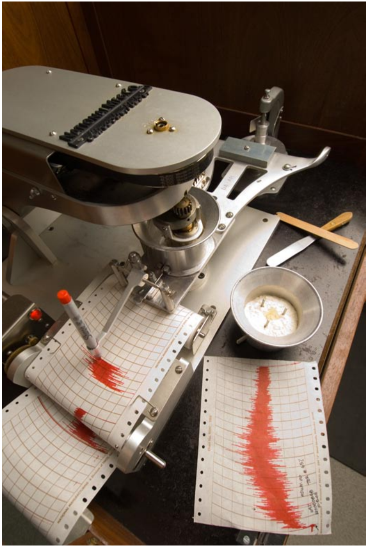






[Department of Food Science, University of Wisconsin. Photos by J-LT.]

the mixograph (2)



The mixograph measures the resistance of the dough to the pin motion.

This is graphed to determine properties of the dough, such as water absorption and 'peak time.'

[Wheat and Flour Testing Methods: A Guide to Understanding Wheat and Flour Quality]

taffy pullers and mixing

play movie

play movie

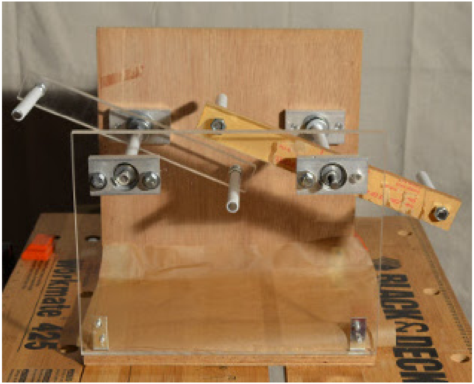
[Boyland, P. L., Aref, H., & Stremler, M. A. (2000). *J. Fluid Mech.* **403**, 277–304; Simulations by M. D. Finn, S. E. Tumasz, and J-LT.]

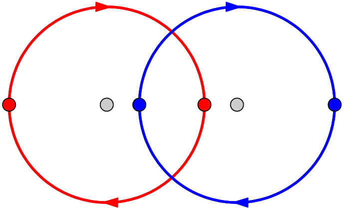
building a mixing device out of Legos

play movie

let's try our hand at this

Six-rod design with undergrad Alex Flanagan:

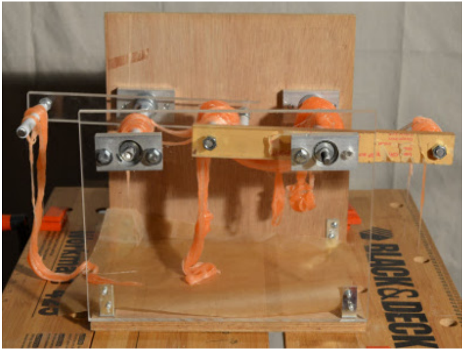


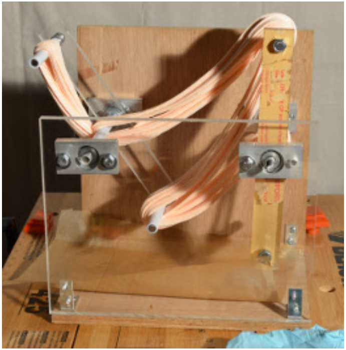


The software tools allow us to rapidly try designs. This one is simple and has huge growth (13.9 vs 5.8 for the standard pullers).

making taffy is hard

Early efforts yielded mixed results: . . . but eventually we got better at it

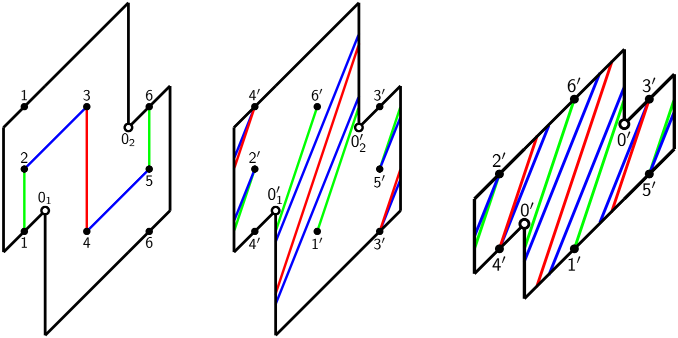




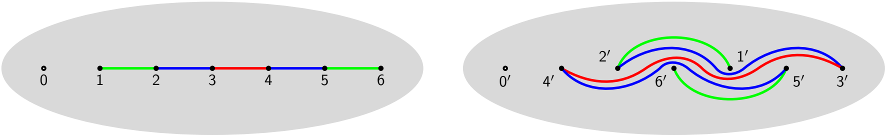
play movie

(BTW: The physics of candy making is fascinating. . .)

six-pronged puller: mathematical construction



$$\phi(x) = \begin{pmatrix} -1 & -1 \\ -2 & -3 \end{pmatrix} \cdot x$$



there is a deeper point here

- My real interest is in fluid mixing, in particular of viscous substances.
- Mixing is very important in many industries, including pharmaceuticals.
- Mixing is a combinatorial process, akin to shuffling.
- The taffy designs also pop up in ‘serious’ chemical mixers.
- The ‘topological dynamics’ approach pioneered by mathematicians allows us to understand these rod motions in great detail, and to design better devices.
- Pinnacle of my math career: reported on in the Food Network.


food network

The Infinite Perfection of Taffy Pulling

More posts from Amy Reiter.

Tags: All Posts, news

references

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