pseudo-Anosovs in the real world

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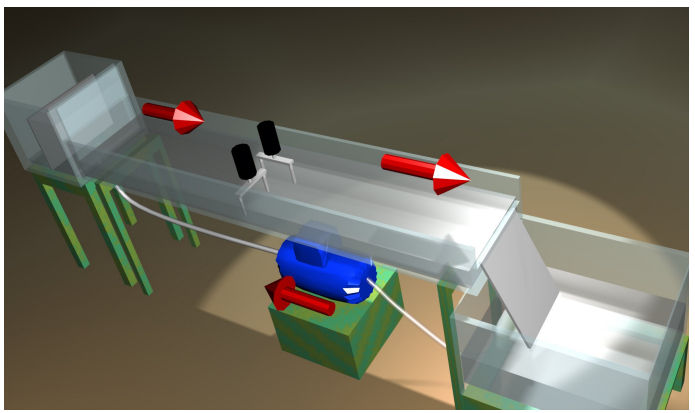
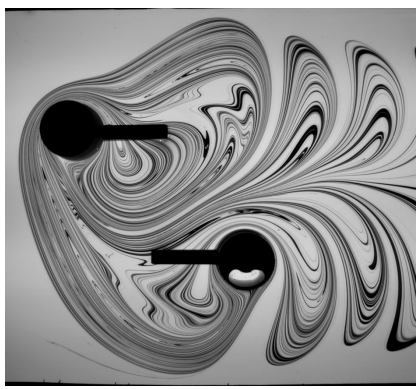
Stirring and Mixing of Viscous Fluids

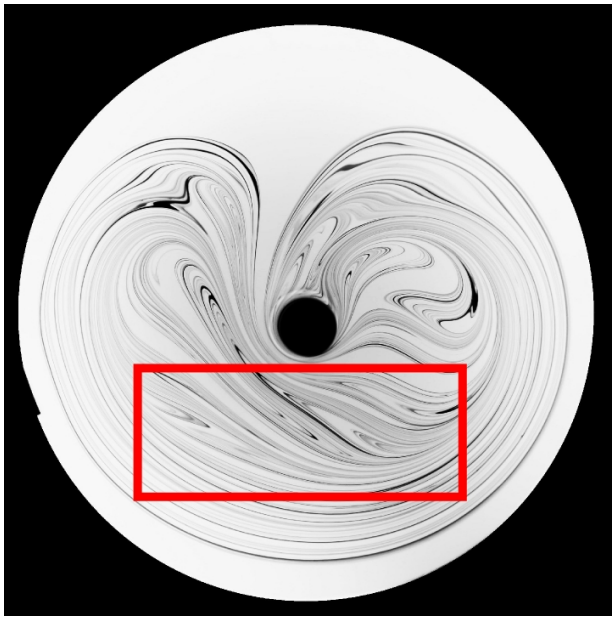
- Viscous flows $\Rightarrow$ no turbulence! (laminar)
- Open and closed systems
- Active (rods) and passive



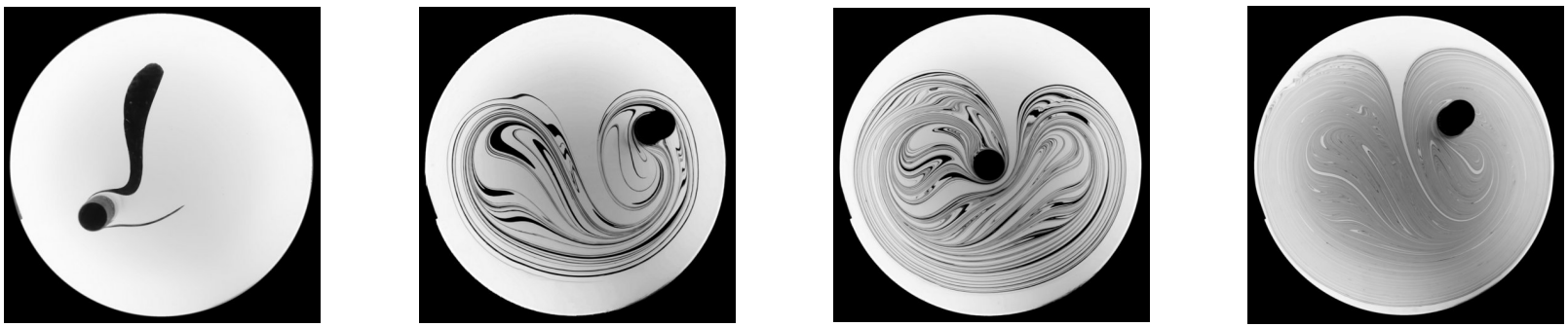


Understand the mechanisms involved.
Characterize and optimize the efficiency of mixing.

The Figure-Eight Stirring Protocol



- Circular container of viscous fluid (sugar syrup);
- A rod is moved slowly in a 'figure-eight' pattern;
- Gradients are created by stretching and folding, the signature of chaos.



[movie 1] Experiments by E. Gouillart and O. Dauchot (CEA Saclay). [snapshots]

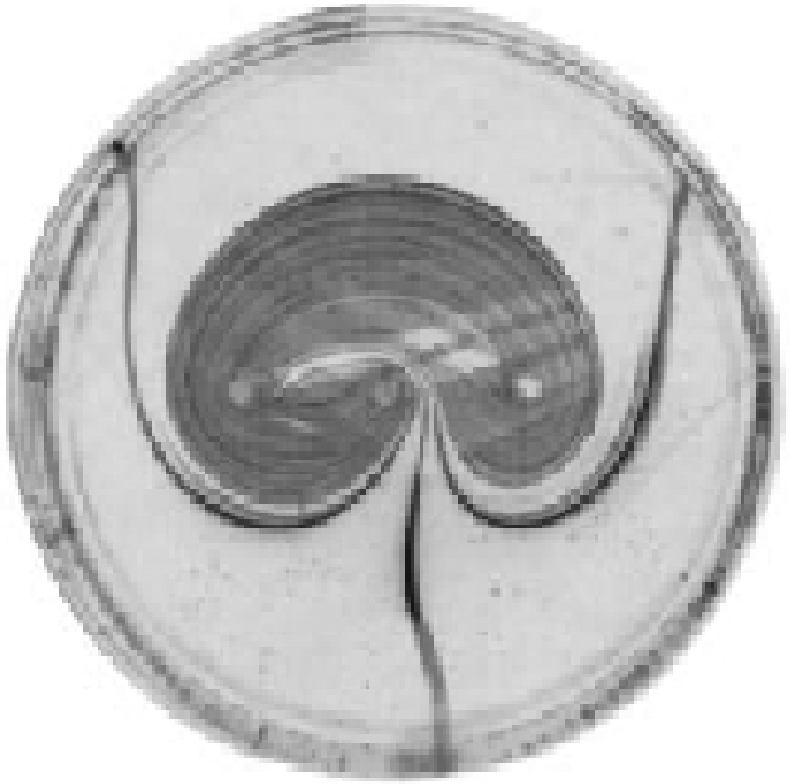
The Taffy Puller

This may not look like it has much to do with stirring, but notice how the taffy is stretched and folded exponentially.

Often the hydrodynamics are less important than the precise nature of the rod motion!

[movie 2]

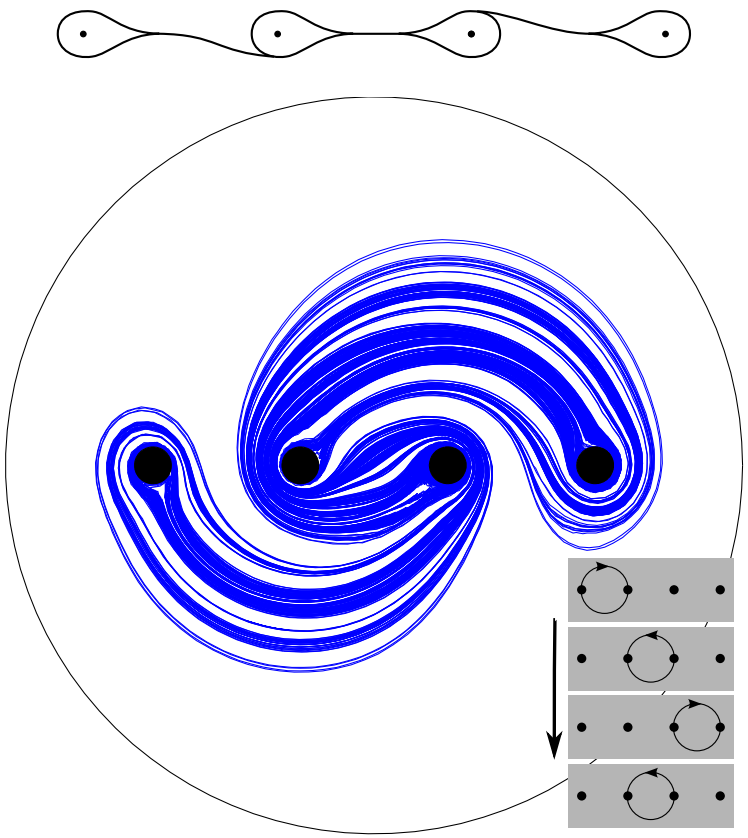
Experiment of Boyland, Aref, & Stremler



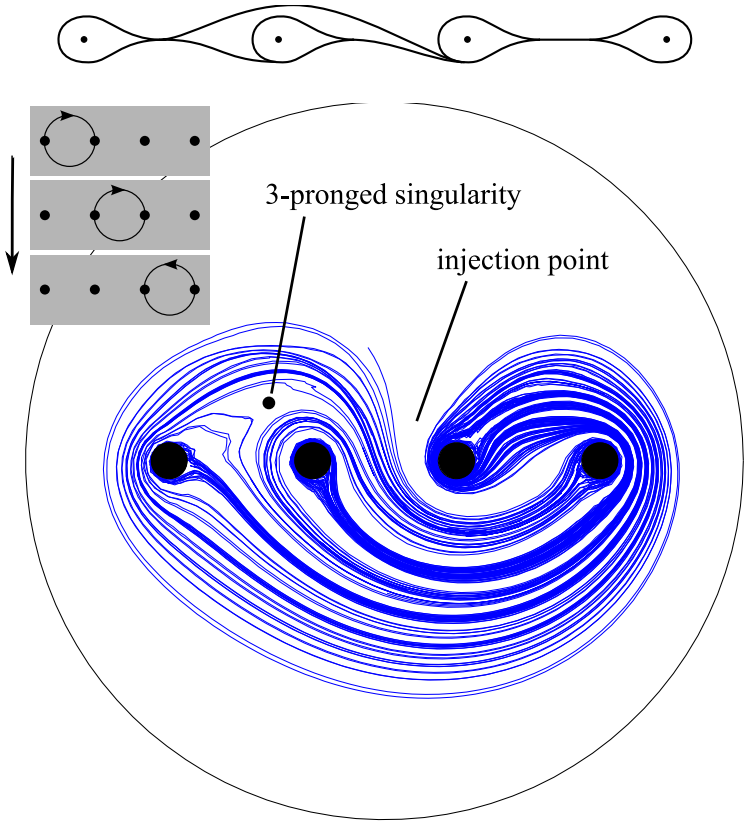
[movie 3] [movie 4] [movie 5]

[P. L. Boyland, H. Aref, and M. A. Stremler, *J. Fluid Mech.* **403**, 277 (2000)]

Two types of stirring protocols for 4 rods



2 injection points

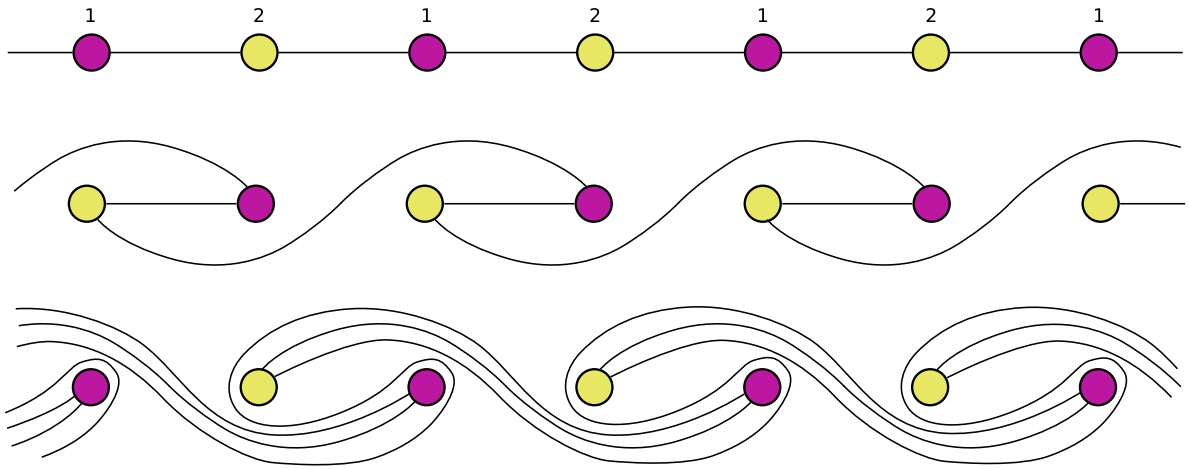


1 injection pt, 1 3-prong sing.

Topological index formulas allow us to classify foliations, and thus stirring protocols (Thiffeault et al., 2008).

Optimization

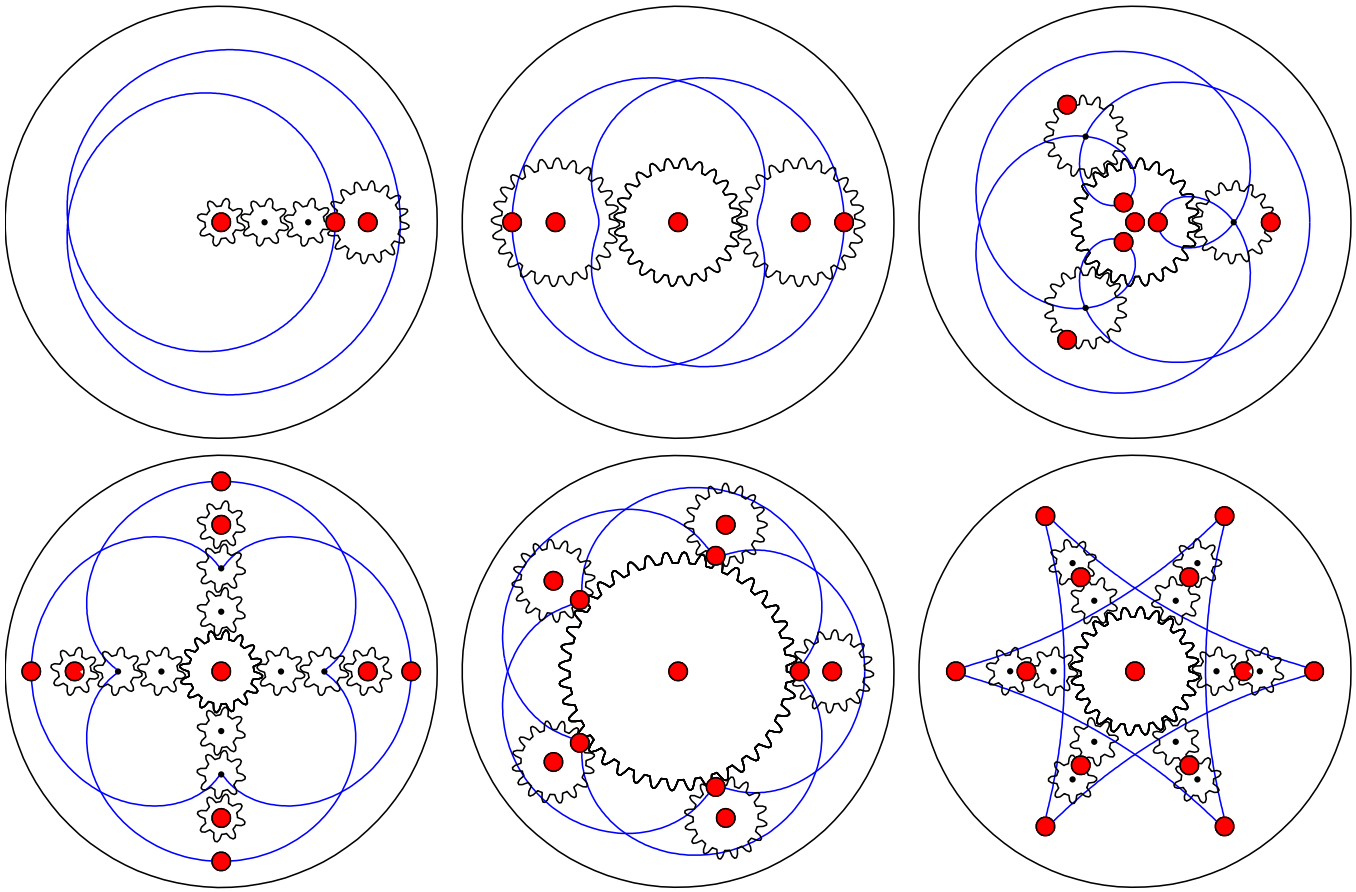
- Consider periodic lattice of rods.
- Move all the rods such that they execute $\sigma_1 \, \sigma_2^{-1}$ with their neighbor (Boyland et al., 2000; Thiffeault & Finn, 2006).



- The dilatation per period is χ^2, where $\chi = 1 + \sqrt{2}$ is the Silver Ratio!
- This is optimal for a periodic lattice of two rods (Follows from D'Alessandro et al. (1999)).

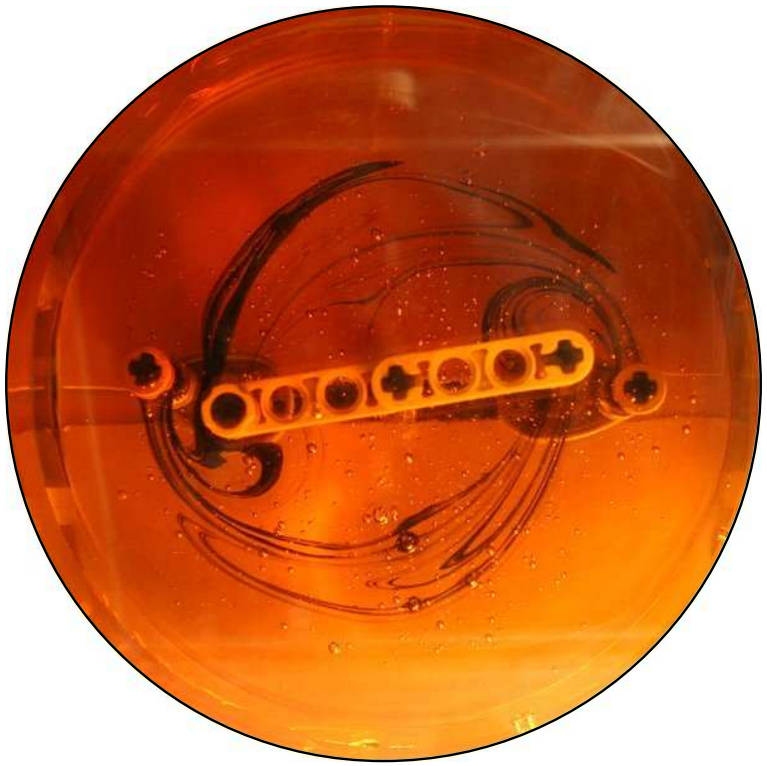
Silver Mixers!

- The designs with dilatation given by the silver ratio can be realized with simple gears.
- All the rods move at once: very efficient.



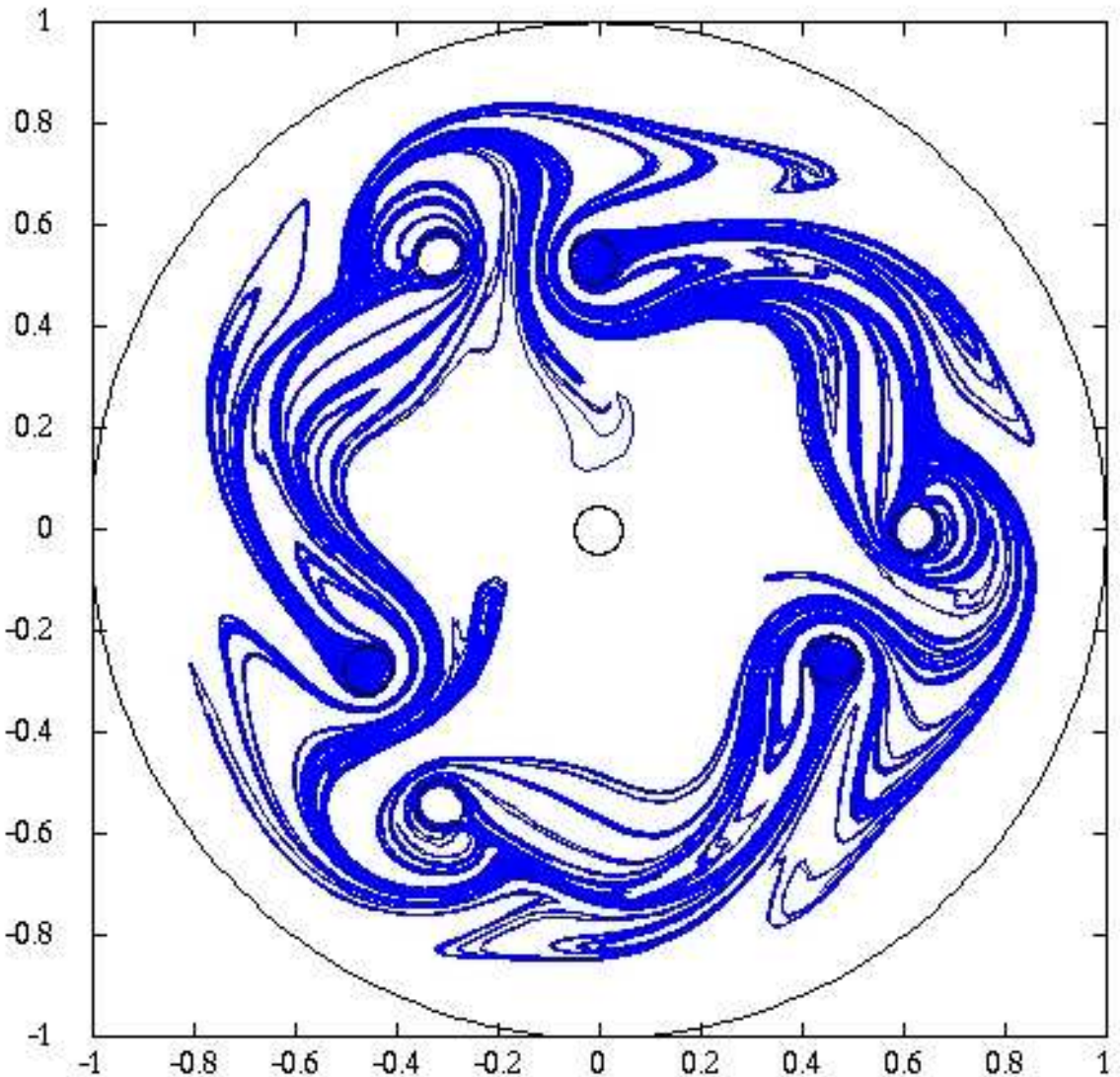
[movie 6]

Four Rods



[movie 7] [movie 8]

Six Rods



[movie 9]

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