Topological mixing of viscous fluids

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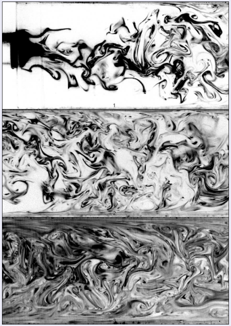
Mixing viscous fluids

Which flows "mix well" ?

Turbulent flows



[Villermaux 2005]

- Atmospheric flows
- Pipe flows, ...

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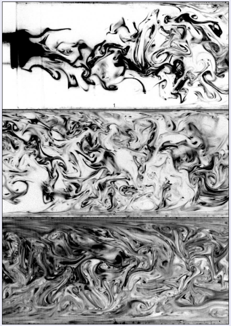
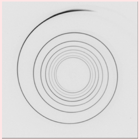


[Villermaux 2005]

- Atmospheric flows
- Pipe flows, . . .

What about low-Reynolds flows ?

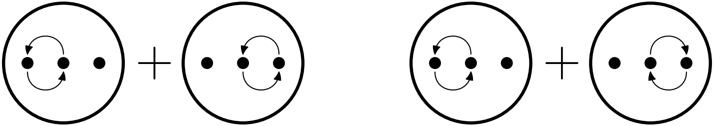
- Simple flows : bad mixing
  [Villermaux 2003]



- Chaotic advection

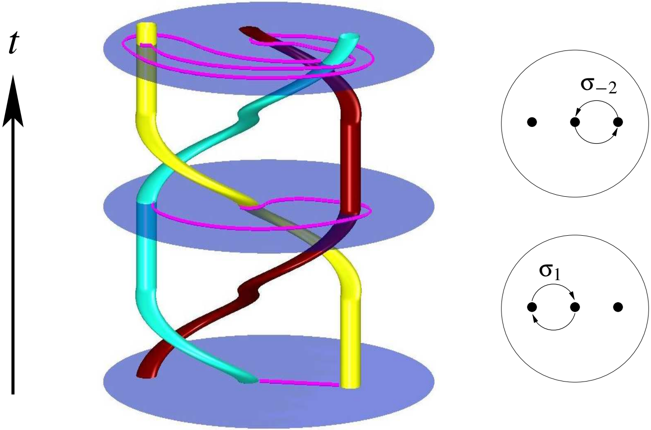
- Food processing, polymers processing, . . .

Experiment of Boyland, Aref, & Stremler



[P. L. Boyland, H. Aref, and M. A. Stremler, *J. Fluid Mech.* **403**, 277 (2000)]

The connection with braids



Spatio-temporal representation of the rod's trajectories = braid !

A universal lower bound on mixing

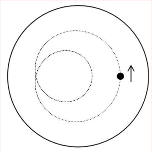
- h_{flow} : topological entropy of the flow, characterizes the exponential stretching of a material line in a chaotic flow.
- h_{braid} : topological entropy of a braid, how "entangled" is the braid ? $h_{\text{braid}} = 0$ for less than 3 rods.
- $h_{\text{flow}} \geq h_{\text{braid}}$
- Thurston–Nielsen theory : braid of rod trajectories restricts class of kinematically-allowable flows.

Topology of the rods' movement ⇒ lower bound on the stretching exponent of the material line (good mixing criterion)
Universal lower bound (does not depend on specific properties of the fluid or on details of the rods trajectories).

Topological chaos with only one rod !

[E. Gouillart, J.-L. Thiffeault, M. Finn, *Phys. Rev. E*, **73**, 036311, 2006]

One-rod protocols



Only one rod ⇒ trivial braid

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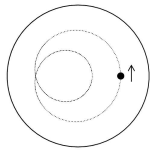
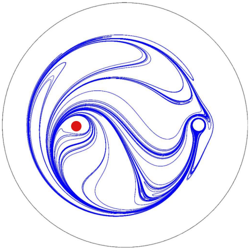
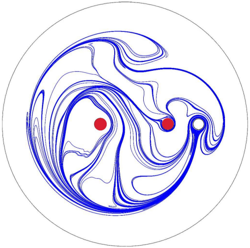


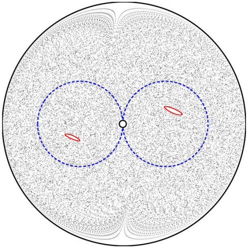
Only one rod ⇒ trivial braid
However, material lines are stretched exponentially ! Similarities with the same protocol with fixed rods.





"Ghost rods" ?

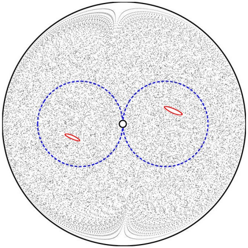
Evidence for ghost rods



Poincaré section :

- One big chaotic region
- Two elliptical islands

Evidence for ghost rods



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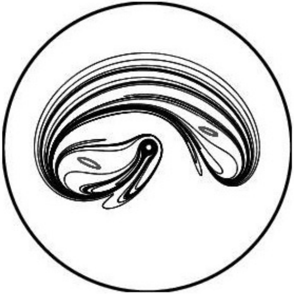


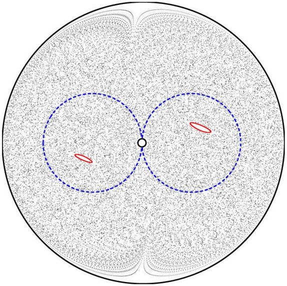
Material lines wrap around the islands.

Evidence for ghost rods

We have detected ghost rods in experiments !

What are ghost rods ?

Elliptical islands



Period-1 elliptical islands

What are ghost rods ?

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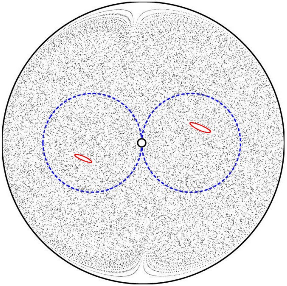
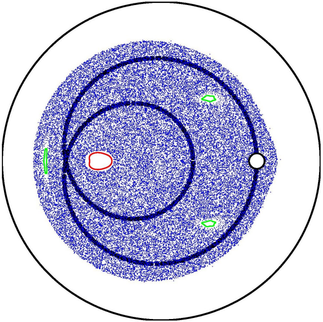


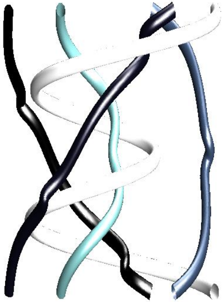
Period-1 elliptical islands

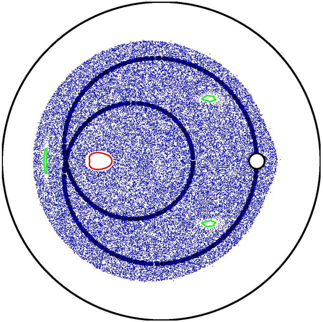


… or elliptical islands of greater period

What are ghost rods ?

Elliptical islands





Elliptical islands are ghost rods.

Efficiency of ghost rods

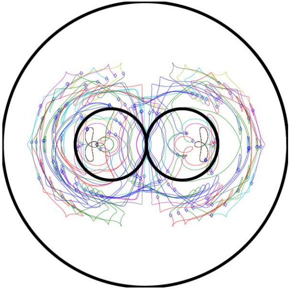
Mixing is more efficient with ghost rods !

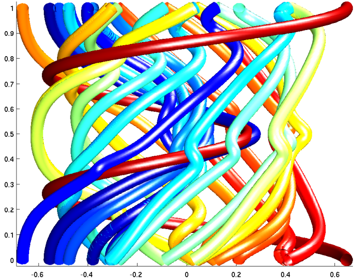
- Greater "topological entropy" (stretching of lines)
- The variance of the concentration PDF decays faster

What are ghost rods ?

All periodic points

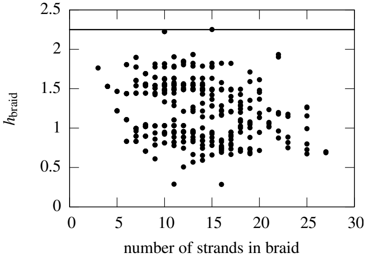
Chaotic flow $\Rightarrow$ infinity of unstable periodic orbits (UPO)





All periodic points are ghost rods.

Chaos means topological chaos !



Theorem

[Katok 1980, Boyland 1994]

There exists a sequence of periodic orbits of the flow whose entropies converge to the topological entropy of the flow h_{flow} $\Rightarrow$ ghost rods account for all the "chaoticity" of the flow.

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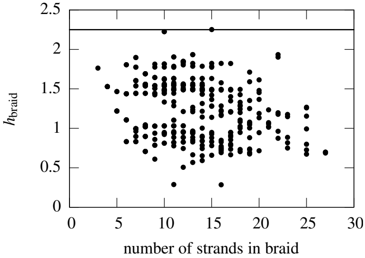

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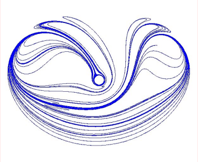
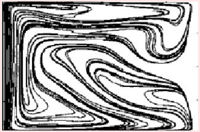
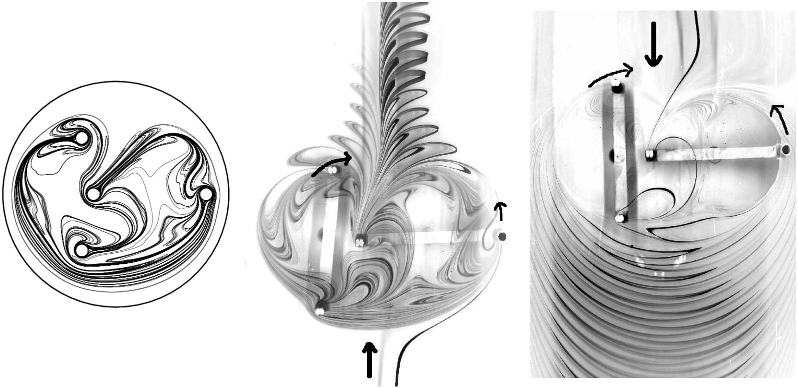


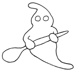


What about open flows ?



Conclusions

Ghost rods



- All periodic structures of a flow = topological obstacles for material lines.
  Example : elliptical island.
- Ghost rods : an original way to characterize chaos in 2D flows (through braids built on trajectories of periodic points).
  $\Rightarrow$ better understanding of chaotic mixing : fluid is mixed thanks to the braiding of material lines by ghost rods.

Conclusions

Many issues to investigate...

- Application to open flows ?
- Some ghost rods are more "efficient" than other : do they have a physical meaning ?
- These topological arguments must be compared to more "natural" mixing indices which directly caracterize homogenization.
- Optimization of braids, periodic domains, etc.